Lg MAGNITUDE CORRECTION FOR THE CENTRAL MISSISSIPPI VALLEY SEISMIC NETWORK*

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ABSTRACT

Using earthquakes that occurred in the region of the Saint Louis University Central Mississippi Valley Seismic Network, the observed Lg-wave amplitude magnitude is modeled as

$$M = S + R + D$$

where S is the source term, R is a station correction, and D is a distance correction. Observations of 458 earthquakes, recorded by similar instruments, peak magnification at about 10 Hz, in the three years period from 1982 to 1984 were fit to this model. The results indicate some important features.

Assuming a coefficient of anelastic attenuation of $\gamma = 0.003km^{-1}$, the distance corrections increase as distance increased. This indicates that a smaller gamma value should be used in the magnitude estimate. The distance correction can be dropped if a $\gamma = 0.0004km^{-1}$ is used. The station corrections reveal station site effects. The stations located in the Embayment need a negative value to correct observed magnitude, whereas, a positive correction is required for stations installed in the Upland.

INTRODUCTION

In the time domain, Lg-phase amplitude data are assumed to satisfied the relation (Ewing et al., 1957)

$$A = A_o \Delta^{-1/3} (R_o \sin\Delta^\circ)^{-1/2} e^{(-\gamma\Delta)}$$

where A is the observed amplitude at epicentral distance Δ(km), A_o is a constant for a given frequency and is related to the source spectral level, and γ is the coefficient of anelastic attenuation. The term $(R_o \sin\Delta^\circ)^{-1/2}$ represents the amplitude decay due to geometrical spreading, Δ° is the epicentral distance in degrees and R_o is the radius of the earth. The term $\Delta^{-1/3}$ represents the decrease in amplitude due to dispersion, since the Lg-wave is as-

* 收稿日期：75年4月15日，送審日期：75年4月21日
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sumed to be a higher-mode surface wave Airy-phase traveling with group velocity of 3.5 km/sec for a continental path (Nuttli, 1973). The term $e^{-\gamma\Delta}$ accounts for frequency-dependent absorption. The parameter γ is frequency dependent, and is related to the specific quality factor, Q, by

$$\gamma = \pi f / QU$$

Where U is the group velocity of 3.5km/sec for Lg-wave, and f is the frequency of the wave.

If the epicentral distance is less than 25°, the geometrical spreading factor can be simplified to

$$(R_o \sin\Delta^\circ)^{-1/2} = \Delta^{-1/2}$$

Therefore, the Lg-amplitude equation becomes

$$A = A_o \Delta^{-5/6} e^{-\gamma\Delta}$$

The measurement of Lg magnitude was modified by Herrmann and Kijko (1983) as

$$m_{Lg} = 2.94 + 0.833 \log_{10} \left(\frac{\Delta}{10}\right) + 0.4342 \gamma r + \log_{10}(A) \qquad (1)$$

The difference between an Lg-magnitude estimated by a network and by an individual station may be due to site effects at the station, to the radiation pattern of the source, or to a different geometrical spreading relation at short distances. To further understand the variation of magnitude, we propose a model to describe observed magnitude as being comprised of a source term, a station term, and a distance term.

Model definition

The observed magnitude estimate, m_{Lg}, for a given event, i, at a station, j, is modeled as the sum of the source magnitude, S_i, a station correction, R_j, and a distance correction, $D_{k(i,j)}$. It can be expressed as

$$m_{Lg} = S_i + R_j + D_{k(i,j)} \qquad (2)$$

The k (i, j) specifies the distance range between the source i and receiver j. The error, ϵ_{ijk}, between the observed and calculated values of magnitude is

$$\epsilon_{ijk} = m_{Lg} - S_i - R_j - D_{k(i,j)} \qquad (3)$$

We consider N_S events and N_R recording stations. The epicentral distances are divided into N_D different ranges. The unknown values of S, R and D in equation (2) can be determined simultaneously from local network data by means of least-square analysis. Since many data are available, the results can also be used to investigate the effects over small areas.

The simultaneous normal equations can be expressed in matrix form as follows

$$A X = Y$$

where A is a symmetric matrix, X is unknown vector including source term, station correction, and distance correction. Vector Y includes the summation of magnitude with respect to individual event, a particular station or distance range.

Normally X can be solved by applying any inversion technique. Since large data sets are expected to be used in this analysis, the matrix of large rank cannot be inverted directly on a small computer due to the limitation of program size. In order to overcome this difficulty, we suggest using a matrix partitioning technique developed by Herrmann (1981) to solve the matrix inversion.

Usually when modeling physical data, all variables are subject to some constraints so that they are not really independent of each other. Using these constraints, it is possible to eliminate some of the variables, and proceed with a smaller set of independent variables. In our study, the model is an extraordinary case in which the elimination of variables is very inconvenient and undesirable from the point of view of automatic matrix array generation. Instead of using a very complicated computer program, we preferred to use an alternative technique involving Lagrangian multipliers and propose a weighting constraint as

$$\phi(R_1, R_2, \ldots, R_{N_R}) = \sum_j N_{S_j} R_j = 0$$

$$\psi(D_1, D_2, \ldots, D_{N_D}) = \sum_k N_{D_k} D_k = 0$$

where N_{S_j} is the number of events for the j'th station, and N_{D_k} is the number of records in the k'th distance range.

Applying a Lagrangian multiplier to the model established, we obtain

$$d\epsilon^2 + \lambda_1\, d\phi + \lambda_2\, d\psi = 0$$

where λ_1 and λ_2 will be relatively small in the solution.

Model Testing

In order to test if the model established is appropriate and if the program constructed using this model is correct, we simulated a set of data of five earthquakes, two distance ranges and five stations (receivers). Using the same notation mentioned above, N_s is 5, N_R is 5 and N_D is 2. The true values are

$S_1 = 5.0$
$S_2 = 4.0$
$S_3 = 3.0$
$S_4 = 4.5$
$S_5 = 3.5$
$R_1 = 0.2$
$R_2 = 0.1$
$R_3 = 0.0$
$R_4 = -0.1$
$R_5 = -0.2$
$D_1 (\leq 50 \text{ km}) = 0.1$
$D_2 (50 \text{ km} - 100 \text{ km}) = -0.1$

First, we would like to see the importance of the constraints. In this experiment no constraints are imposed. Therefore, the last two columns and the last two rows are not used in the first matrix. Consequently, λ_1 and λ_2 are not needed. The solutions are

EVENT	S	95% confidence interval
1	-14.12	± 0.0003
2	-15.12	± 0.0003
3	-16.12	± 0.0003
4	-14.62	± 0.0003
5	-15.62	± 0.0004

STATION	R	95% confidence interval
Ast	25.60	± 0.0005
Bst	25.50	± 0.0005
Cst	25.40	± 0.0004
Dst	25.30	± 0.0006
Est	25.20	± 0.0001

DISTANCE	D	95% confidence interval
0-50	-6.18	± 0.0007
50-100	-6.38	± 0.0001

From the mathematic view point, the small 95% confidence interval value indicates that the solutions are reliable. But, as we anticipated, all variables are adjusted relative to each other and the solutions may not have too much physical meaning. The relative differences in the source, station and difference terms reflect those of the model values, but an arbitrary offset of coefficients is present such that the magnitudes computed are correct.

In the second experiment, we use the constraints outlined above. The solutions are very consistent with the expected values. They are listed as follows

EVENT	S	95% confidence interval
1	5.00	± 0.00002
2	4.00	± 0.00002
3	3.00	± 0.00002
4	4.50	± 0.00002
5	3.50	± 0.00003

STATION	R	95% confidence interval
Ast	0.20	± 0.00003
Bst	0.10	± 0.00003
Cst	0.00	± 0.00002
Dst	-0.10	± 0.00002
Est	-0.20	± 0.00003

DISTANCE	D	95% confidence interval
0-50	0.10	± 0.00001
50-100	-0.10	± 0.00002

λ_1 = −8.0977e-08 λ_2 = −2.7675e-07

It is worth to note that excellent agreement was found since the original model had $\Sigma R=0$, $\Sigma D=0$. If the ΣR are not in fact zero, then this technique will never be able to determine this unless there is an additional independent constraint, such as a statement that the R at one station is by definition zero. To further understand the model and also to simulate closely the actual data set, random noise was added to each simulated observation. All noise was required to be less than half of the station correction and the summation of noise is zero, which obeys the constraints we used above. The solutions are

EVENT	S	95% confidence interval
1	5.00	± 0.009
2	3.99	± 0.010
3	2.99	± 0.011
4	4.50	± 0.011
5	3.50	± 0.013

STATION	R	95% confidence interval
Ast	0.20	± 0.011
Bst	0.12	± 0.012
Cst	-0.01	± 0.010
Dst	-0.08	± 0.010
Est	-0.21	± 0.014

DISTANCE	D	95% confidence interval
0-50	0.10	± 0.053
50-100	-0.10	± 0.010

where

λ_1 = 2.2769e-08 λ_2 = 1.0942e-07

The increase of the 95% confidence interval value is caused by the random noise added. However, the solutions are still very stable. This test further confirms the applicability of our model and computer programs.

DATA AND RESULTS

The data for this study consist of seismograms of 458 earthquakes (Figure 1) occurring in the period 1 January 1982 to 31 December, 1984, in the area of the Central Mississippi Valley Seismic Network, operated by Saint Louis University.

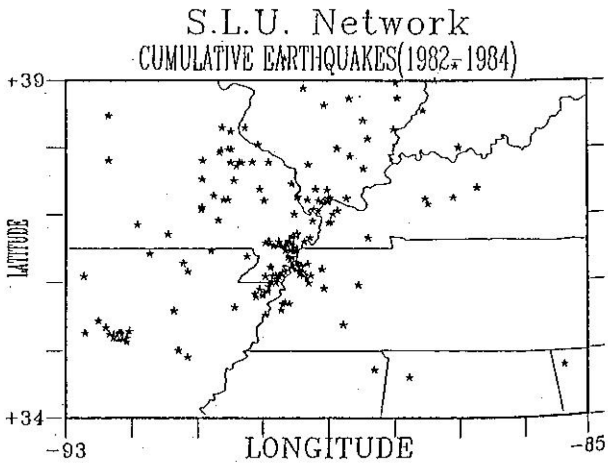

Figure 1. A map of earthquakes in the period 1982 to 1984 used for this study. Total number of events is 458.

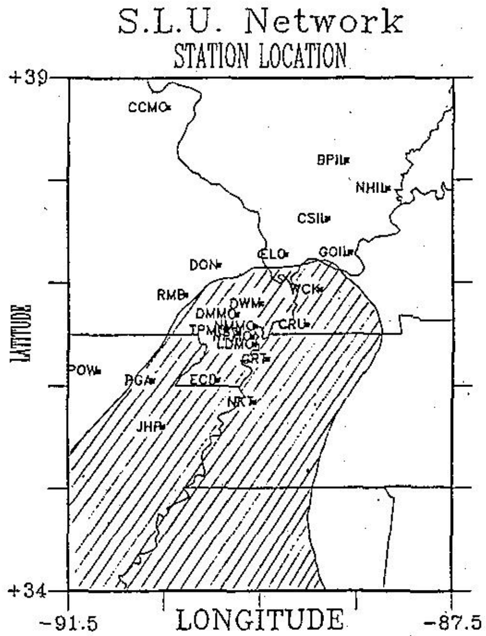

Figure 2. Map of the SLU network. 23 stations used in this study are shown in the figure. The region of the Mississippi Embayment is shaded.

The network consists of 24 stations in the Mississippi Embayment region, 7 stations in the Upland region, and 9 stations in southeastern Illinois. Figure 2 shows a map of the 23 stations of the SLU Seismic Network used in this study. The region of the Embayment is indicated by the shading (Hadley and Devine, 1974). The 23 stations used in this study were restricted to those with similar instrument response (peak magnification at 10 Hz). The coordinates of these stations as well as their net amplifier gains between seismometer and the computer are given in Table 1. The normalized instrument response is plotted in Figure 3.

The observed magnitudes were calculated by using equation (1) where reduced ground amplitude (A) is provided by the SLU network and the coefficient of anelastic attenuation (γ) is 0.003 km^{-1}

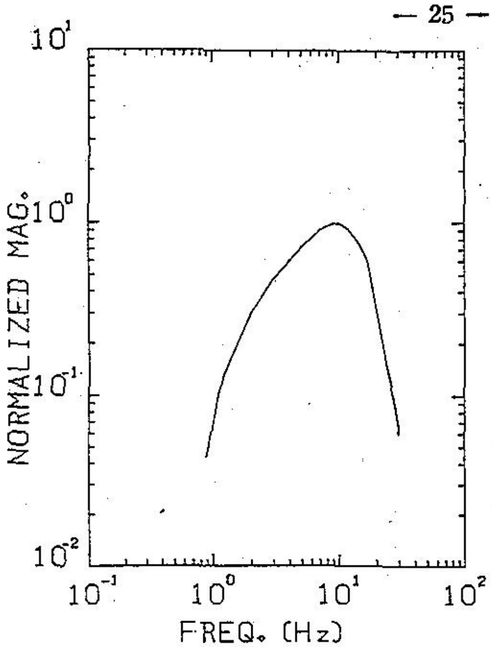

Figure 3. Normalized instrument response.

TABLE 1

Saint Louis University Network

Station Code	Latitude N°	Longitude W°	Magnification (db)
LDMO	36.411	89.563	68
POW	36.152	91.185	74
RMB	36.888	90.278	68
LST	36.523	89.731	56
CRU	36.595	89.020	62
DMMO	36.704	89.745	68
DON	37.176	89.933	68
DWM	36.805	89.490	56
ECD	36.060	89.940	62
ELC	37.285	89.227	74
GRT	36.264	89.420	62
NKT	35.850	89.554	62
PGA	36.060	90.620	62
JHP	35.605	90.510	62
WCK	36.934	88.874	68
CSIL	37.632	88.790	78
NRMO	36.487	89.588	62
NMMO	36.588	89.552	56
TPMO	36.540	89.852	68
BPIL	38.200	88.600	72
GOIL	37.300	88.560	60
CCMO	38.720	90.470	68
NHIL	37.930	88.170	66

(Dwyer, *et al.*, 1983). The determination of reduced ground amplitude in the SLU network is either done by selecting a peak amplitude with period at about 0.1 second from digital data on a Tectronix terminal or by selecting a sustained amplitude (third largest) from develocorder manually, then corrected for the corresponding instrument response at the observed frequency. The difference in methodology, peak versus third largest peak, will not introduce a major error in the magnitude estimate since logarithms of amplitudes are taken.

The distance range are separated by 20 km interval for epicentral distances less than 200 km. For records at distance greater than 200 km, a 50 km interval was

TABLE 2

Distance Correction Terms

Distance		D		95% Confidence interval	N
0-20	D1	-0.022	±	0.010	382
20-40	D2	-0.069	±	0.020	333
40-60	D3	-0.100	±	0.050	129
60-80	D4	-0.128	±	0.044	160
80-100	D5	-0.082	±	0.033	222
100-120	D6	0.040	±	0.010	186
120-140	D7	0.052	±	0.012	116
140-160	D8	0.161	±	0.030	151
160-180	D9	0.194	±	0.037	57
180-200	D10	0.121	±	0.020	31
200-250	D11	0.248	±	0.067	79
250-300	D12	0.248	±	0.089	66
300-500	D13	0.383	±	0.135	38

TABLE 3

Station Correction Terms

Station	R	95% Confidence interval	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13
LDMO	-0.03	0.005	76	19	1	2	1	4	1	0	1	0	0	2	0
POW	-0.05	0.004	1	0	1	1	3	11	41	74	6	3	13	2	2
RMB	0.04	0.003	0	0	11	45	48	74	10	7	4	3	17	11	1
LST	-0.01	0.005	73	24	6	0	4	1	1	0	1	0	0	5	2
CRU	0.08	0.007	0	1	30	11	5	1	1	2	0	0	2	0	1
DMMO	0.02	0.004	18	42	16	11	9	4	1	0	1	0	0	9	1
DON	-0.03	0.004	0	1	7	31	39	19	11	7	4	4	4	12	2
DWM	0.16	0.010	4	3	4	6	1	1	0	0	0	0	0	1	1
ECD	-0.09	0.008	9	6	8	6	0	0	4	4	0	0	2	0	1
ELC	-0.27	0.03	37	51	4	9	45	22	20	22	5	2	8	2	11
GRT	0.07	0.005	26	25	7	3	1	5	2	0	2	1	2	4	1
NKT	0.22	0.05	1	15	13	8	9	0	1	14	1	4	7	4	1
PGA	0.13	0.08	0	0	1	3	3	10	1	1	20	4	0	1	1
NMMO	0.29	0.07	50	3	1	1	2	0	0	0	0	0	0	0	0
TPMO	-0.11	0.07	8	25	3	0	3	5	0	1	0	0	0	3	1
NRMO	0.22	0.05	75	24	4	2	3	2	1	0	1	1	0	2	0
JHP	0.26	0.11	0	0	1	0	3	5	3	6	2	0	2	0	0
WCK	0.00	0.004	3	74	5	16	35	13	8	3	3	4	5	1	7
CSIL	-0.59	0.12	0	0	1	2	0	0	2	4	2	2	0	1	2
BPIL	-0.49	0.18	1	0	0	1	1	2	1	0	0	0	2	0	0
CCMO	0.15	0.09	0	0	3	0	4	2	1	1	1	2	12	5	1
NHIL	-0.06	0.018	0	0	1	1	3	2	0	0	0	0	1	0	1
GOIL	0.02	0.077	0	20	2	1	0	3	8	5	3	1	2	1	1

$\lambda_1 = 3.58783E\text{-}05$, $\lambda_2 = 3.59471E\text{-}05$

used and for distances greater than 300 km, a single correction was used. In general, we have 458 source terms, 23 stations, and 13 distance ranges in the analysis, and 1950 amplitude magnitude observations. The solutions are listed in the order of distance corrections and station corrections in Table 2 and Table 3.

DISCUSSION AND CONCLUSION

Another way to express the suitability of a solution is by the distribution of residuals. Figure 4 plots an example of the magnitude residual histogram for the station LDMO. The top histogram in the figure shows a magnitude residual histogram before the magnitude correction (m_{Lg}), where a magnitude residual histogram after the magnitude correction (S) is given below it. In general, a more concentrated pattern of residuals about zero is found in the second plot. This implies an improvement in magnitude calculation by use of the correction terms. The other 22 stations all have similar results.

The similar detection condition for all the stations (see some examples in Figure 5) indicates our results might not be biased by different recording usability between stations.

The station corrections are plotted at the corresponding station location on a map in Figure 6. It is easy to see that the station corrections in the Embayment region (shaded area) are small negative or are positive values, except at the station TPMO (–0.12). On the other hand, the stations in the Upland region are mostly negative or are small positive values, except at the station CCMO (0.14).

To see if the distance corrections are contaminating the station corrections, we fixed the distance corrections at zero,

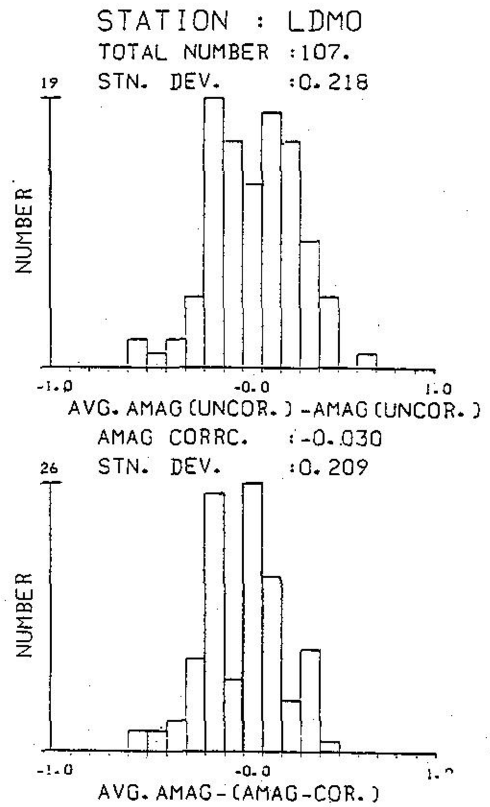

Figure 4. Amplitude magnitude residual histograms for the station LDMO. The top histogram shows the residual distribution before correction applied whereas the lower histogram shows the residual distribution after station correction and distance correction applied.

even though the F-test analysis indicated that such a correction is needed. The station corrections from this model, model 2, are listed in Table 4 with the above results for comparison. In the last column of Table 4, the 'E' represents station located in Embayment area and 'U' in Upland. Examining this table, we found similar results from both models. This tells us the model stability and also implies that station corrections are not related to the specifies of the correction due to the distance term in the magnitude formula.

Distance corrections obtained in the previous section increase with increasing distance. This tendency indicates that

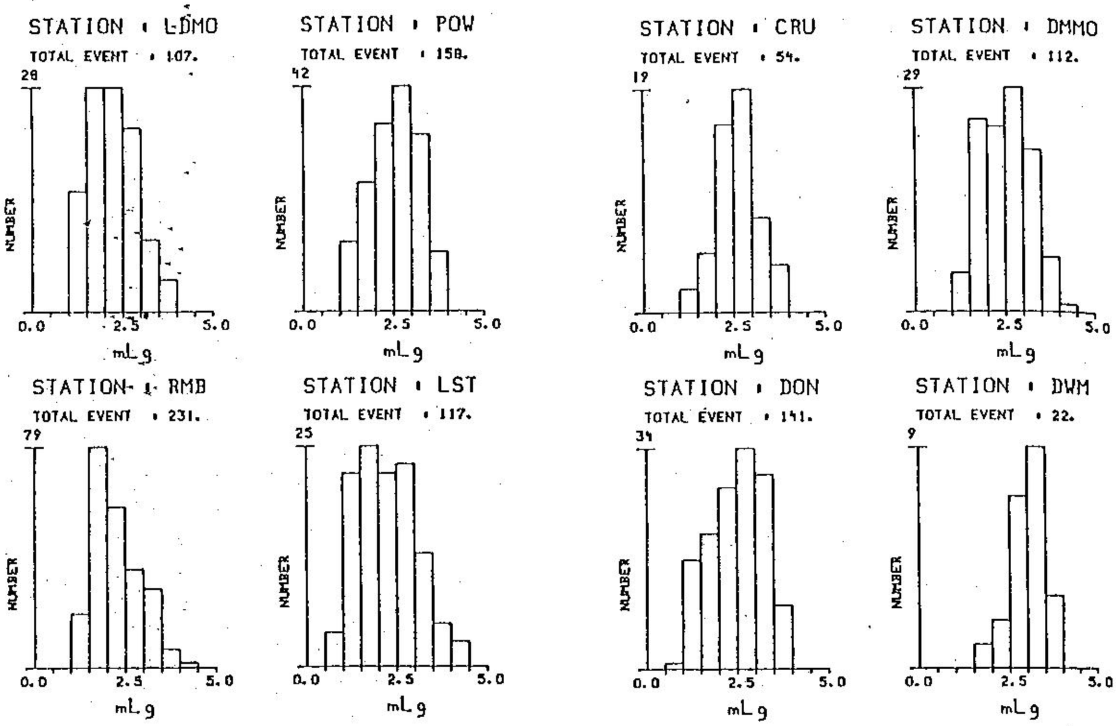

Figure 5. Histogram showing number of detected event at some stations as a function of magnitude.

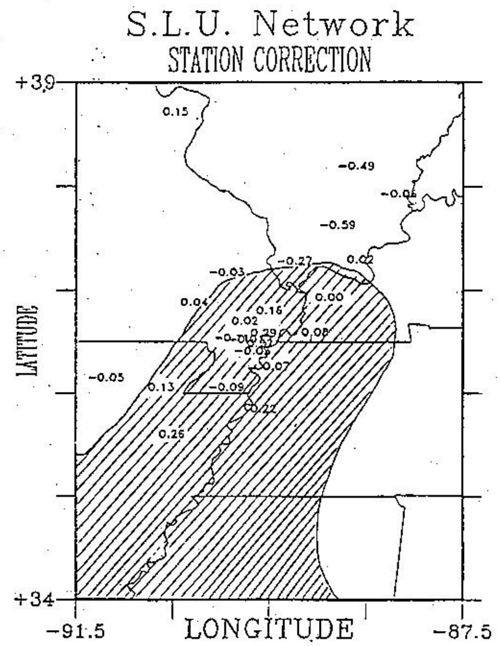

Figure 6. The amplitude magnitude station correction and the corresponding station location. The Embayment region, a thicker sediment near the surface, is shaded. In general, the station in Embayment region has negative station correction. On the other hand, stations in the Upland region have positive value.

TABLE 4
Station Amplitude Magnitude Correction

Station	Model 1[1]	Model 2[2]	Model 3[3]	Region
LDMO	-0.03	-0.04	-0.03	E
POW	-0.05	0.04	-0.06	U
RMB	0.04	0.04	0.04	U
LST	-0.01	-0.02	-0.01	E
CRU	0.08	0.03	0.08	E
DMMO	0.02	-0.02	0.02	E
DON	-0.03	-0.04	-0.03	U
DWM	0.16	0.12	0.16	E
ECD	-0.09	-0.13	-0.09	E
ELC	-0.27	-0.25	-0.26	U
GRT	0.07	0.05	0.07	E
NKT	0.22	0.20	0.22	E
PGA	0.13	0.14	0.13	E
JHP	0.26	0.24	0.26	E
WCK	0.00	-0.03	0.00	E
CSIL	-0.59	-0.52	-0.60	U
NRMO	0.22	0.21	0.23	E
NMMO	0.29	0.28	0.29	E
TPMO	-0.11	-0.14	-0.12	E
BPIL	-0.49	-0.46	-0.49	U
GOIL	0.02	0.03	0.02	U
CCMO	0.15	0.25	0.14	U
NHIL	-0.06	-0.11	-0.04	U

1. Model 1: Original model
2. Model 2: No distance term model
3. Model 3: γ = 0.0004km^{-1} model

E: Embayment, U: Upland

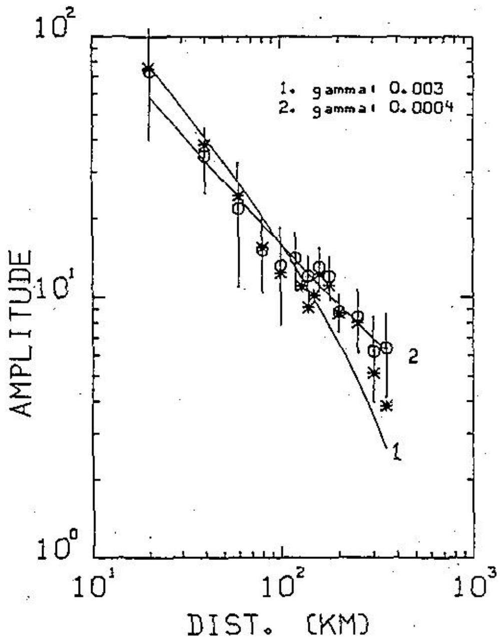

Figure 7. The distance correction is related to different gamma values. Curve 1 shows the amplitude decay with gamma = 0.003 km^{-1} and curve 2 uses a smaller gamma value (0.0004 km^{-1}). The data points are obtained by using the theoretical Lg attenuation relation and the distance corrections.

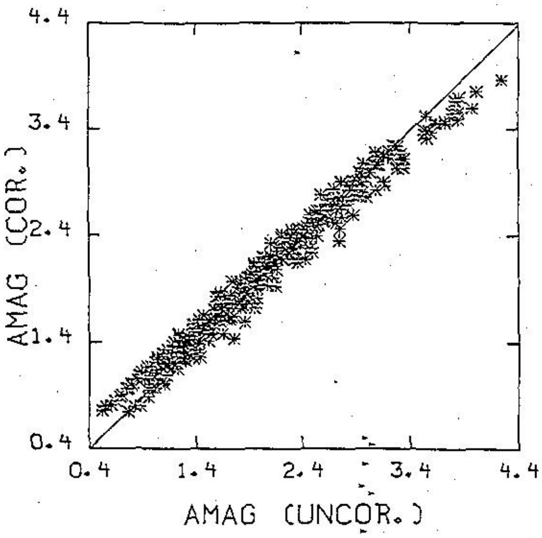

Figure 8. Comparison of amplitude magnitudes before and after applying correction. The straight line has a slope of one.

either the geometrical factor or the anelastic attenuation used is improper. In order to avoid the contamination caused by station corrections included in the original model, the data were reanalyzed with all station corrections fixed to be zero. The results are plotted in Figure 7. The solid line (1) is the theoretical amplitude when the geometrical factor is 5/6 and gamma value is 0.003 km^{-1}. The circles are amplitudes after a distance correction which are obtained from the original model, and the error bar indicates 95% confidence interval. The stars are amplitude after a distance correction obtained from the model 2 with station corrections equal to zero. The results from both of these model indicate the need of distance correction to revise the magnitude formula equation 4 for the distance range (< 500 km).

The shape of curve 1 in Figure 7 indicates that use of a gamma value smaller than 0.003 km^{-1} might be appropriate. Using the circles in Figure 7, a simple regression resulted in $\gamma = 0.0004\ km^{-1}$. The solid line (2) in Figure 7 corresponds to the Lg-airy phase amplitude decay with $\gamma = 0.0004\ km^{-1}$. Using this gamma value, the station corrections are recalculated. The results are listed on the third column of the Table 4. The distance corrections are very small and can be neglected. For easy usage of these results we suggest the magnitude determination as

$$m_{Lg} = 2.94 + 0.833 \log_{10} \left(\frac{\Delta}{10}\right) + 0.4342\gamma\tau + \log_{10}(A) - R$$

with $\gamma = 0.0004\ km^{-1}$. The term R is station correction and is listed in the third column of Table 4.

To see the improvement as the magnitude corrections obtained in this study are applied in the magnitude estimate, we plot uncorrected magnitude versus corrected magnitude in Figure 8. Offsets with respect to a line of slope 1 occur at both sides. This tells us that Lg magnitudes of large events are essentially overestimated by 0.2-0.3 magnitude units without the magnitude correction, whereas small events are underestimated by about 0.2 magnitude units. The fact that the large magnitude were somewhat overestimated is related to the larger gamma value used in the previous analysis.

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美國中西部密西西比地震網的Lg規模修正

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摘　　要

本文利用美國中西部，密西西比地震網的地震資料分析，研究有關Lg 規模修正。依Lg波振幅計算所得規模可以下列計算模式來討論：

$$M = S + R + D$$

在此模式中，S是震源項，R是測站修正項，D則是距離修正項。本文挑選發生在一九八二年一月至一九八四年十二月的地震共四百五十八個，使用其中具有相似反應曲線的儀器所記錄的資料，套入此模式中，得到許多重要的特性。

在推算過程中，如假設非彈性衰減 (anelastic attenuation) 係數，γ(gamma) 為 $0.003 km^{-1}$，所得的 Lg 規模距離修正值是隨着距離的增加而增加。此結果反應出在規模的估算中，須採用一較小的 γ (gamma)值。將 γ(gamma) 值換成 $0.0004km^{-1}$，距離修正值即可省略。模式計算結果並顯示測站修正值可以反應出測站附近的地質效應。位於密西西比河河套的測站需要一個負值來修正。相對的，如測站位置在較硬的岩盤時，則需要一個正值來修正。