潮位資料之時頻分析
A New Spectrogram of Tide Wave

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Abstract

The tide wave data of Chenggong harbor at Taitung, Taiwan, which covers the range from August 1st, 2002 to July 31th , 2004, is examined by a new time frequency transform. The transform involves two steps: one evaluates the Fourier sine spectrum; and the other imposes a window to the spectrum at a given frequency whose inverse Fourier transform is corresponding to the real part of the spectrogram. The frequencies and amplitudes of dominant modes are approximately equal to that evaluated by the harmonic analysis except some scatterings on the spectral domain. From the spectrogram, variations of both amplitude and frequency of dominant tide waves are clearly inspected. For those waves closely related to the moon period, which have the wavelength of 7, 14, 28, 42, and 56 days, the non-stationary frequency and amplitude variations are captured. Nonlinear behaviors are also founded in the waves with 6 and 8 hours wavelength. These waves are referred to the reflection of Kurosio current from the continental shelf of Eurasian plate. These new information show that the new spectrogram has the potential to start a new study upon the ocean long waves.

Key word: New spectrogram, tide wave decomposition

1. Introduction

For time series data, as noted by Farge [1], a time frequency analyzing tool can capture the variation of spectrum with respect to time which reflects the involved details and mechanism(s). Therefore, spectrograms generated by time frequency transforms, such as short time Fourier, Gabor and wavelet transforms, are widely applied [2]. Although distributions of original Wigner-Ville distribution and Cohen class are also successful transforms [2,3], the related methodology will go beyond the scope of this study and it is not discussed here.

In several previous studies [4-7], it is seen that both Gabor and continuous wavelet (Morlet) transforms give detailed information by embedding a window to weight the data string on the time domain. These studies also show that a window on the time domain results in a band-pass filtered spectrum. Based on this fact, a new time frequency transformation was proposed in Ref.[8]. In this study, the application of this transform to do the wave decomposition of the tide wave is shown.

2. Theoretical Development

2.1 Intrinsic Error of the Discrete Fourier Expansion

From the functional analysis, it is well known that to represent a data string, one can employ either strong or weak projection. Consider a discrete data string, the dimensions of the complete function space is finite. Suppose that the eigen functions space is complete. The error of a strong eigen expansion formula at every point must be zero. On the other hand, the error of a weak eigen function expansion formula will be nonzero at least at one point. A typical weak projection example is the discrete Fourier expansion. In fact, because of the periodic properties of sine and cosine function, a strong projection via the Fourier expansion is achieved only if all the periodic conditions of $y, y', ..., y^{(N-1)}$ are satisfied, where y is the dependent variables and N is the data size. In other words, all the Fourier coefficients contain certain error because of the non-periodic properties of the data string. Moreover, the non-sinusoidal part of a data string will introduce the well known Direct Current (DC) contamination. As a consequence, the minor modes may be seriously faded and dominant modes may be slight distorted.

It is well known that the Fourier expansion is an approximation in the least squares sense which is a non-robust estimation as mentioned in Ref.[9]. For example, suppose that a data string y_i is approximated by a function $f(x_i, a_0, a_1, ..., a_M)$, where $a_0, a_1, ..., a_M$ are parameters to minimize a error measure function $\rho(z)$ where $z = [y_i - f(x_i, a_0, ..., a_M)]/\sigma$ and σ is the error deviation parameter. From the probability point of view, the minimization procedure is correspond- ing to

$$\sum_{i=0}^{m}\frac{1}{\sigma_i}\frac{d\rho(z)}{dz}\frac{\partial f(x_i)}{\partial a_j}=0,\ j=0,1,\ldots,M \qquad (1)$$

in which the variation $d\rho(z)/dz$ is the weight of the problem. A least square method assumes that the errors are distributed as the Gaussian normal distribution

$$\text{Prob}\{\, y_i - f(x_i,p)\,\} \approx \exp(-|\frac{y_i - f(x_i,2)}{\sigma_i}|^2) \qquad (2)$$

Consequently, $\rho(z)=z^2/2$ and the weight is z such that a least square method uses a larger and larger weight as the error increases. It indicates that the resulting estimation will be biased by minor or isolated data points scattering from the main trend. In other words, for the Fourier expansion, once the expansion range is slightly changed or a few data points are removed, the resulting spectrum will deviated from the original one that can not be ignored. Therefore, most people have the experience that different FFT version gives different spectrum but their main features are similar to each other.

2.2 A New Spectrogram Generator

Under the assumption that time series data has no discontinuous component, the procedure of generating a new spectrogram involves the following steps [8].

1. Apply the iterative filter of Ref.[10] to remove the data's non-sinusoidal part whose contribution to the spectrum is also named as the Direct Current (DC) contamination.
2. Use the strategy of Ref.[11] to generate a Fourier sine spectrum whose spectrum error is small.
3. Add a window to get a band-pass limited spectrum for a given frequency. The corresponding real part of transform is the inverse FFT.
4. The real part's Hilbert transform is the corresponding amplitude.
5. Plot the two-dimensional spectrogram.

For the sake of completeness, the theoretical content of Ref.[8] is briefly restated below.

2.3. Iterative Filter with Diffusive Property [10]

Consider a data string, $y_0, y_1, \ldots, y_N$ and expand it as follows.

$$y(t)=\sum_{n=0}^{N-1}(b_n - jc_n)\exp[j2\pi t/\lambda_n] \qquad (3)$$

If the Gaussian smoothing method is employed to smooth the data, it can be shown numerically that it is a diffusive smoothing. But the transition zone of this low-pass filter is too wide. In order to narrow down the transition zone, an iterative filter based on Gaussian smoothing was developed in Ref.[10]. The iteration procedure smoothes the remaining high frequency part repeatedly. If the iteration stops at the $m-$th step, the final high frequency part $y'(t)$ is the desired short wave part. The final smooth part is

$$\bar{y}(t)=y(t)-y'(t)$$
$$\approx \sum_{n=0}^{N-1}[1-A_n]^m (b_n - jc_n)\exp[j2\pi t/\lambda_n] \qquad (4).$$
$$0 \le A_n(\sigma/\lambda_n) \approx \exp[-2\pi^2\sigma^2/\lambda_n^2] \le 1$$

Like the Gausssian smoothing method, the iterative filter is also diffusive. Supposing a data string has a frequency gap in the range of $\bar{\lambda}_0 < \lambda < \bar{\lambda}_1$ within which all modes are not important, both m and σ can be solved by the simultaneous equations with factor $[1-A(\sigma/\bar{\lambda})]^m$ equal to almost 0 and 1, respectively. If there is no such gap, the above procedure can be considered as initial work. By extracting the data associated with those modes with $\lambda < \bar{\lambda}_1$ from the spectrum of the remaining high frequency part, a sharp filter cutting at λ_1 is obtained [12-14].

2.4. Spectrum with small error

In Ref.[4-8,11-12], the above mentioned iterative filter is employed to remove the non-sinusoidal and low frequency parts. For the remaining high frequency part, the simple strategy of FFT [11] generates a Fourier sine spectrum with very small spectrum and DC errors. The strategy includes: find zeros at two ends; remove data beyond zeros; redistribute data so that the total number of points is equal to an integer power of 2; do an odd function mapping to ensure periodicity; follow, finally, with an FFT algorithm. Since the resulting spectrum employs all the necessary periodic conditions and is referred to as a strong projection technique.

2.5. New Time Frequency Transform [8]

Assume that the high frequency part, $y'(t)$, is expressed in the form of Eq.(1). Any time-frequency transform [1-8] can be applied to generate a spectrogram without DC error. For example, the Gabor transform gives [4]

$$G(f,\tau)=1/\sqrt{a}\int_{-\infty}^{\infty} y'(t)e^{-2j\pi f(t-\tau)}e^{-(t-\tau)^2/(2a^2)}dt$$
$$\approx \sqrt{\pi a/2}\sum_{n=0}^{\infty}(b_n - jc_n)e^{2j\pi f_n \tau}e^{-4\pi^2 a^2 (f_n - f)} \qquad (5)$$

where a is the scale function. The Gaussian window imposed on the time domain clearly results in imposing a corresponding Gaussian window on the spectrum domain. The Morlet transform and all enhanced Gabor and Morlet transforms have similar mappings of windows between time and frequency domains [5-8].

Based on this fact, a Gaussian window is imposed on the spectrum of $y'(t)$ so that a band-passed data string corresponding to f_k can be obtained [8]

$$y_k(\tau)=\sum_{n=0}^{N-1}(b_n - jc_n)e^{j2\pi\tau/\lambda_n}e^{-(n-k)^2/(2c^2)} \qquad (6)$$

where the last exponential term is the Gaussian window with window size c. Now $y_k(t)$ is considered as the mode or real part corresponding to f_n. The amplitude of $y_k(t)$ is evaluated either by the original [10] or modified Hilbert transform [8]. The Hilbert transform is fast but has the penalty of convolution error due to end effect. When the modified Hilbert transform is employed, the spectrum, by the proper feeding of 0's to the function, say $e^{j2\pi f_n t}/t$, is unavailable now so that it should be done on time domain. Finally, the spectrogram is obtained by scanning all frequencies.

Along a $f = f_k$ line, the real part of the spectrogram is a mode with a Gaussain window size characterized by parameter c on spectrum. For the problems of turbulent flow data, modes generated by different values of c correspond to different physical meanings and should be carefully addressed.

3. Results and Discussions

The test case is the tide data of Chenggong harbor in Taitung, Taiwan. The sampling period covers from Aug. 1st, 2002 to July 31th, 2004 with a sampling rate 10 points per hour. Since there are several missing data period, the data repairing technique of Ref [15] is employed to repair the data and the data reduction is made to pick one from every 10 data points. Figures 1 shows the original and smooth part (including the non-sinusoidal part). The smooth part shows that the water level rises in summer time and drops in winter time. There was a distinct wavelike change between September 2003 and February 2004, probably due to a significant typhoon passed nearby in early September and strong clod waves occurred in the afterward winter. Figures 2a through 2c are the spectrum estimated by the Fourier transform with (referred as Fourier 1 in Table-1) and without (referred as Fourier 2) the smooth part and the Fourier sine spectrum, respectively. The main features of these figures are similar to each other except that the smooth part introduces a DC contamination to Fig.1a. However, the Fourier spectrum of the high frequency part is slightly different from the Fourier sine spectrum because the periodic conditions of $y',...,y^{(N-1)}$ can not be exactly satisfied. Table 1 shows that comparison of three dominant modes between these spectrums and result of the harmonic analysis. For the M2 wave, the result of the Fourier transform with the smooth part is the closest one with respect to that of the harmonic analysis. For S2 and K2 waves, results of the Fourier transform without the smooth part are the closest approaches to that of the harmonic analysis. Note that these coincided cases resolve the dominant waves by single modes. As to the rest cases, results of the Fourier and Fourier sine spectrums use more than one mode to resolve dominant modes. In fact, the scattering of the spectrum reflects the corresponding variations of amplitude and frequency on the time domain. The following spectrograms will confirm variations of these modes. Note that the scattering among these Fourier expansion is principally induced by the non-robustness of the least squares method.

Figure 3a to 3b are spectrograms of the daily tide wave components generated by the proposed time frequency transform, Morlet, and Gabor transforms, respectively, where the Gabor transform employs the Gaussian window width of $a = 60$ days and is almost the best result The main features of three figures are similar to each other, but Fig.3a achieves the best information resolution. Obviously, all modes have the amplitude and frequency variations. There are apparent signals shown in day 200-400, could be related to frequent typhoons occurred in March-September 2003.

4. Conclusions

The new Fourier sine spectrum and spectrogram are successively employed to study the tide wave data in a two years period of the Chenggong harbor of Taitung, Taiwan. From the resulting spectrum and spectrogram, many new information, which can not be provided by current methods, are grasped. Many further studies upon this and related issues are on the way.

5. Acknowledgement

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6. References

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TABLE 1 Dominant Modes Comparison

	Harmonic Analysis	Fourier 1	Fourier 2	Fourier sine
M2 freq..	0.080511	0.080511	0.080477	0.080477
Amp.	0.4876	0.470578	0.22302	0.089492
freq.			0.080534	0.080505
amp.			0.363446	0.41759
freq.			0.080591	0.080534
amp.			0.097440	0.158714
S2 freq	0.083333	0.083305	0.083329	0.083300
amp.	0.2018	0.128476	0.195147	0.023453
freq.		0.083362		0.083329
amp.		0.126192		0.192099
freq.				0.083357
amp.				0.021181
K2 freq.	.08356149	0.083533	0.083557	0.083500
amp.	.0533	0.044122	0.059294	0.012998
freq.		0.083590		0.083528
amp.		0.0368983		0.026761
freq.				0.083557
amp.				0.030251
freq.				0.083585
amp.				0.041296

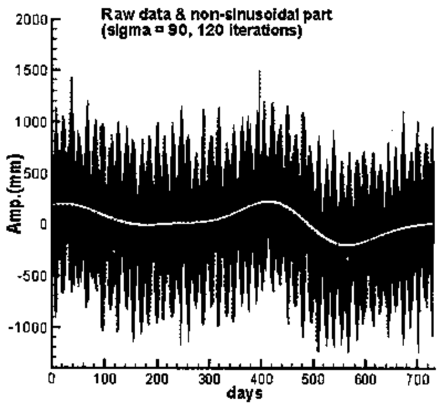
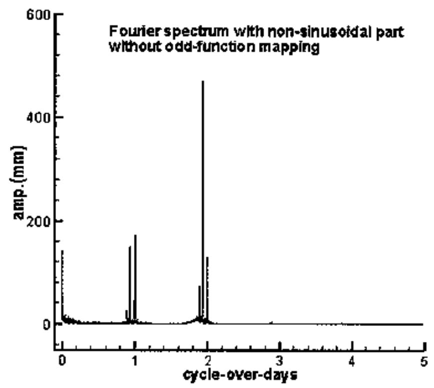
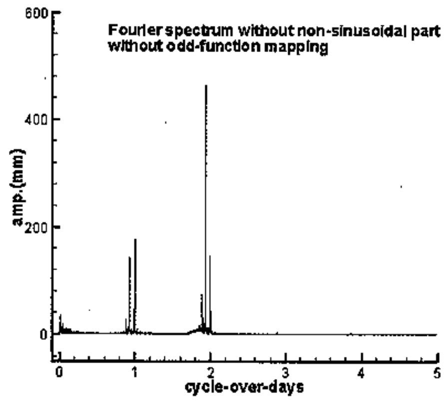


Fig.1 The raw and non-sinusoidal data of Chenggong harbor's tide wave: thin line is the raw data and heavy line is the non-sinusoidal part.

(a)



(b)



(c)

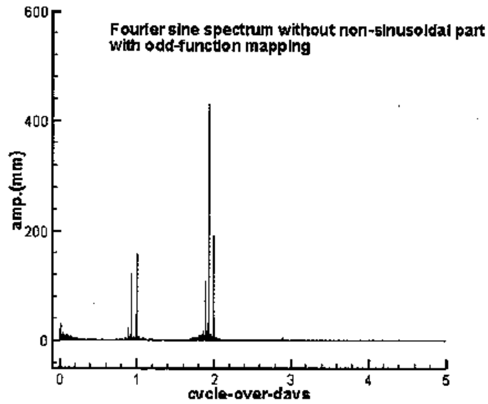
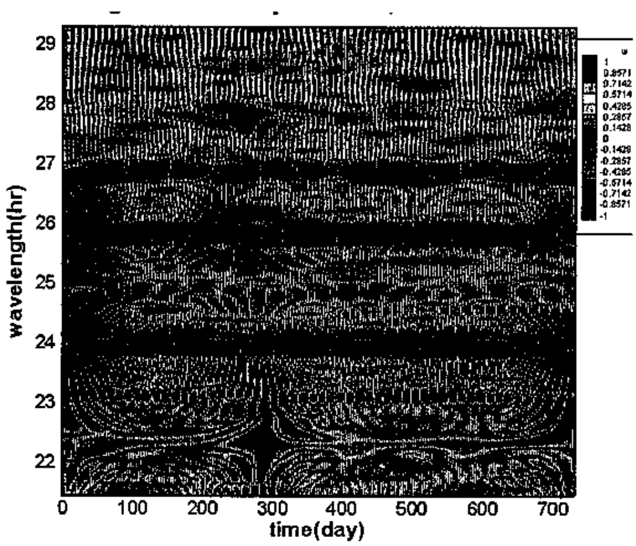
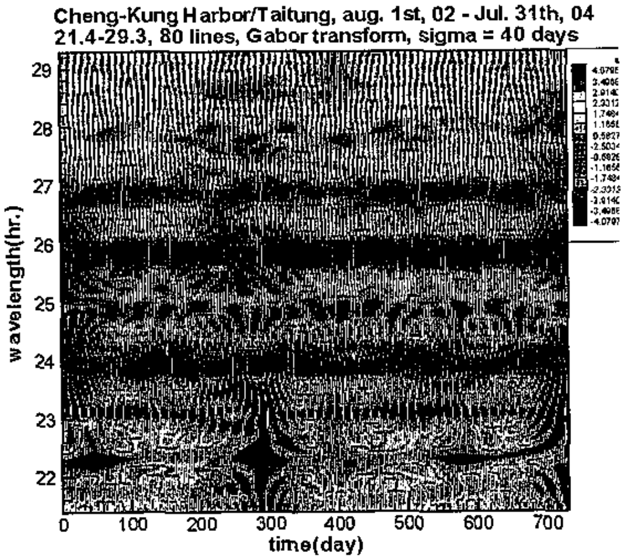


Fig.2 The spectrum of the tide wave data: (a) Fourier spectrum with the non-sinusoidal part; (b) Fourier spectrum without the non-sinusoidal part; and (c) Fourier sine spectrum without the non-sinusoidal part.

(a)



(b)



(c)

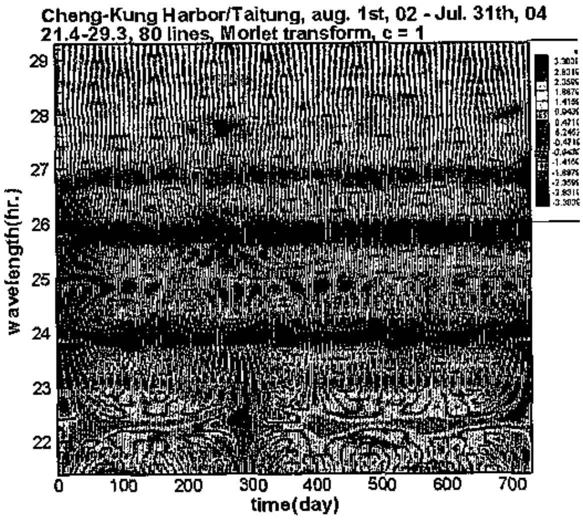


Fig.3 The spectrogram plot around the 24 hour tide wave: (a) generated by the proposed transform; (b) by the Gabor with the best window width of $a = 40$ days; and (c) by the Morlet transform.