THE PERFORMANCE OF HYBRID SIGMA-THETA NCEP GFS

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1. Introduction

This extended abstract can be the part two of the extended abstract of Juang 2005 of CWB (Central Weather Bureau) annual meeting in last year. As mentioned, the generalized vertical hybrid coordinates for atmospheric modeling has been developed, such as Simmons and Burridge 1981; Zhu et al 1992; Konor and Arakawa1997; Johnson and Yuan 1998; Benjamin et al 2004. With generalized hybrid vertical coordinates, the atmospheric model can be integrated along any different types of coordinate surfaces. The coordinates near surface and lower atmosphere used to apply terrain following sigma coordinates, but over the upper atmosphere, they had better to compute on quasi-horizontal such as pressure surfaces or isentropic surfaces to reduce the numerical errors due to estimated vertical motions. The combination of these coordinates as hybrid coordinates can take advantage of individual type of the coordinate surfaces for numerical purpose.

We have implemented our own approached generalized vertical hybrid coordinates into NCEP GFS. To reduce the possible complication to the model and its related and downstream software, we have done an incremental implementation, as mentioned in last year's extended abstract. Such that, all prognostic variables as what we used are used as the prognostic variables; spectral computation in horizontal and finite difference in vertical are kept in this implementation. Again, as mentioned, due to the hybrid coordinate equation set is different from what we have, new discretization in vertical to satisfy energy and angular momentum conservations is utilized. The matrixes used for semi-implicit time integration have to be modified due to different vertical discretization in hybrid coordinates.

Quasi-parallel runs with different hybrid coordinates have been done for a period of more than half year. This report describes a short summary of current result of parallel runs of NCEP hybrid-theta GFS. Section 2 lists the completed set of all discretized equations pseudo spherical coordinate. Section 3 discusses the vertical flux with specific vertical coordinates, which can be used as pure sigma, sigma-pressure and sigma-theta. Section 4 illustrates parallel runs and their performance and discussion is in the last section.

2. Discretized hydrostatic system on spherical and generalized hybrid coordinates

The discretized primitive hydrostatic system on spherical coordinates in horizontal and generalized hybrid coordinate in vertical can be written as following Juang 2005, they are

$$\frac{\partial u_k^*}{\partial t} = -m^2 u_k^* \frac{\partial u_k^*}{a\partial\lambda} - m^2 v_k^* \frac{\partial u_k^*}{a\partial\varphi} - \frac{1}{2}\left\langle \underline{\underline{\left(\dot{\zeta}\frac{\partial\hat{p}}{\partial\zeta}\right)_k \frac{u_{k-1}^* - u_k^*}{\hat{p}_k - \hat{p}_{k+1}}}} + \overline{\overline{\left(\dot{\zeta}\frac{\partial\hat{p}}{\partial\zeta}\right)_{k+1} \frac{u_k^* - u_{k+1}^*}{\hat{p}_k - \hat{p}_{k+1}}}} \right\rangle$$
$$- \frac{R_d T_{vk}}{\hat{p}_k + \hat{p}_{k+1}}\left[\frac{\partial \hat{p}_k + \hat{p}_{k+1}}{a\partial\lambda}\right] - \frac{\partial\Phi_s}{a\partial\lambda}$$
$$-\sum_{i=1}^{k-1}\frac{R_d T_{vi}}{\hat{p}_i + \hat{p}_{i+1}}\left[\frac{\partial\hat{p}_i - \hat{p}_{i+1}}{a\partial\lambda} - \frac{\hat{p}_i - \hat{p}_{i+1}}{\hat{p}_i + \hat{p}_{i+1}}\frac{\partial\hat{p}_i + \hat{p}_{i+1}}{a\partial\lambda}\right]$$
$$-\sum_{i=1}^{k}\frac{R_d T_{vi}}{\hat{p}_i + \hat{p}_{i+1}}\left[\frac{\partial\hat{p}_i - \hat{p}_{i+1}}{a\partial\lambda} - \frac{\hat{p}_i - \hat{p}_{i+1}}{\hat{p}_i + \hat{p}_{i+1}}\frac{\partial\hat{p}_i + \hat{p}_{i+1}}{a\partial\lambda}\right]$$
$$-R_d\sum_{i=1}^{k-1}\frac{\hat{p}_i - \hat{p}_{i+1}}{\hat{p}_i + \hat{p}_{i+1}}\frac{\partial T_{vi}}{a\partial\lambda} - R_d\sum_{i=1}^{k}\frac{\hat{p}_i - \hat{p}_{i+1}}{\hat{p}_i + \hat{p}_{i+1}}\frac{\partial T_{vi}}{a\partial\lambda} + f_s v_k^* + F_{u_k^*}$$

$$\frac{\partial v_k^*}{\partial t} = -m^2 u_k^* \frac{\partial v_k^*}{a\partial\lambda} - m^2 v_k^* \frac{\partial v_k^*}{a\partial\varphi} - \frac{1}{2}\left\langle \underline{\underline{\left(\dot{\zeta}\frac{\partial\hat{p}}{\partial\zeta}\right)_k \frac{v_{k-1}^* - v_k^*}{\hat{p}_k - \hat{p}_{k+1}}}} + \overline{\overline{\left(\dot{\zeta}\frac{\partial\hat{p}}{\partial\zeta}\right)_{k+1} \frac{v_k^* - v_{k+1}^*}{\hat{p}_k - \hat{p}_{k+1}}}} \right\rangle$$
$$- \frac{R_d T_{vk}}{\hat{p}_k + \hat{p}_{k+1}}\left[\frac{\partial \hat{p}_k - \hat{p}_{k+1}}{a\partial\varphi}\right] - \frac{\partial\Phi_s}{a\partial\varphi}$$
$$-\sum_{i=1}^{k-1}\frac{R_d T_{vi}}{\hat{p}_i + \hat{p}_{i+1}}\left[\frac{\partial\hat{p}_i - \hat{p}_{i+1}}{a\partial\varphi} - \frac{\hat{p}_i - \hat{p}_{i+1}}{\hat{p}_i + \hat{p}_{i+1}}\frac{\partial\hat{p}_i + \hat{p}_{i+1}}{a\partial\varphi}\right]$$
$$-\sum_{i=1}^{k}\frac{R_d T_{vi}}{\hat{p}_i + \hat{p}_{i+1}}\left[\frac{\partial\hat{p}_i - \hat{p}_{i+1}}{a\partial\varphi} - \frac{\hat{p}_i - \hat{p}_{i+1}}{\hat{p}_i + \hat{p}_{i+1}}\frac{\partial\hat{p}_i + \hat{p}_{i+1}}{a\partial\varphi}\right]$$
$$-R_d\sum_{i=1}^{k-1}\frac{\hat{p}_i - \hat{p}_{i+1}}{\hat{p}_i + \hat{p}_{i+1}}\frac{\partial T_{vi}}{a\partial\varphi} - R_d\sum_{i=1}^{k}\frac{\hat{p}_i - \hat{p}_{i+1}}{\hat{p}_i + \hat{p}_{i+1}}\frac{\partial T_{vi}}{a\partial\varphi} - f_s u_k^* - m^2\frac{s_k^{*2}}{a}\sin\phi + F_{v_k^*}$$

$$\frac{\partial T_{vk}}{\partial t} = -m^2\vec{V}_k \bullet \nabla T_{vk}$$
$$-\left\{ \underline{\underline{\left(\dot{\zeta}\frac{\partial\hat{p}}{\partial\zeta}\right)_k \frac{\alpha_{k-1}T_{vk-1} - \gamma_k T_{vk}}{(\hat{p}_k - \hat{p}_{k+1})}}} + \overline{\overline{\left(\dot{\zeta}\frac{\partial\hat{p}}{\partial\zeta}\right)_{k+1} \frac{\delta_k T_{vk} - \beta_{k+1}T_{vk+1}}{(\hat{p}_k - \hat{p}_{k+1})}}} \right\}$$
$$+\frac{\kappa T_{vk} m^2}{\hat{p}_k + \hat{p}_{k+1}}\left\{ V_k^* \bullet \nabla(\hat{p}_k + \hat{p}_{k+1}) - \sum_{i=k}^{K}\left((\hat{p}_i - \hat{p}_{i+1})D_k^* + V_k^* \bullet \nabla(\hat{p}_i - \hat{p}_{i+1})\right)\right.$$
$$\left. -\sum_{i=k+1}^{K}\left((\hat{p}_i - \hat{p}_{i+1})D_k^* + V_k^* \bullet \nabla(\hat{p}_i - \hat{p}_{i+1})\right)\right\} + F_{T_{vk}}$$

$$\frac{\partial \hat{p}_k}{\partial t}=-m^2\sum_{i=k}^{K}\left((\hat{p}_i-\hat{p}_{i+1})\left(\frac{\partial u_i^*}{a\partial\lambda}+\frac{\partial v_i^*}{a\partial\varphi}\right)+u_i^*\frac{\partial(\hat{p}_i-\hat{p}_{i+1})}{a\partial\lambda}+v_i^*\frac{\partial(\hat{p}_i-\hat{p}_{i+1})}{a\partial\varphi}\right)$$

$$-\left(\dot{\zeta}\frac{\partial\hat{p}}{\partial\zeta}\right)_k$$

$$\frac{\partial q_{i_k}}{\partial t}=-m^2u_k^*\frac{\partial q_{i_k}}{a\partial\lambda}-m^2v_k^*\frac{\partial q_{i_k}}{a\partial\varphi}$$

$$-\frac{1}{2}\left\langle\left(\dot{\zeta}\frac{\partial\hat{p}}{\partial\zeta}\right)_k\frac{q_{i_{k-1}}-q_{i_k}}{\hat{p}_k-\hat{p}_{k+1}}+\left(\dot{\zeta}\frac{\partial\hat{p}}{\partial\zeta}\right)_{k+1}\frac{q_{i_k}-q_{i_{k+1}}}{\hat{p}_k-\hat{p}_{k+1}}\right\rangle+F_{q_n}$$

under the conditions of

$$\left(\dot{\zeta}\frac{\partial\hat{p}}{\partial\zeta}\right)_{K+1}=\left(\dot{\zeta}\frac{\partial\hat{p}}{\partial\zeta}\right)_1=0$$

$$\frac{\partial\hat{p}_{K+1}}{\partial t}=\nabla\hat{p}_{K+1}=0$$

$$\frac{\partial\hat{p}_1}{\partial t}=\frac{\partial p_s}{\partial t}$$

and

$$p_k=\frac{1}{2}(\hat{p}_{k+1}+\hat{p}_k)$$

$$\alpha_{k-1}=\frac{1}{2}\left(\frac{\hat{p}_k+\hat{p}_{k+1}}{\hat{p}_{k-1}+\hat{p}_k}\right)^{\kappa}$$

$$\beta_{k+1}=\frac{1}{2}\left(\frac{\hat{p}_k+\hat{p}_{k+1}}{\hat{p}_{k+1}+\hat{p}_{k+2}}\right)^{\kappa}$$

$$\gamma_k=\frac{1}{2}-\kappa\frac{\hat{p}_k-\hat{p}_{k+1}}{\hat{p}_k+\hat{p}_{k+1}}$$

$$\delta_k=\frac{1}{2}+\kappa\frac{\hat{p}_k-\hat{p}_{k+1}}{\hat{p}_k+\hat{p}_{k+1}}$$

These equations can be closed as the vertical fluxes for all interfaces are given. The vertical fluxes will be given in a general method described in the next section.

3. Specific generalized hybrid coordinates and the solution of vertical fluxes

Up to the previous section, the finite difference equation sets are for generalized hybrid coordinates, which can be used for sigma, sigma-pressure, sigma-theta, and sigma-theta-pressure etc. While a specific coordinate is used, these finite difference equations can be used with some modification. Following can be a specific one for generality covering sigma, pressure and/or isentropic as

$$\hat{p}_k=\hat{A}_k+\hat{B}_kp_s+\hat{C}_k\left(\hat{T}_{vk}/\hat{T}_{0k}\right)^{C_p/R_d}$$

where A, B and C are specified and constant during integration with following known boundary conditions for pressure at top atmosphere and surface, as

$$\hat{A}_{K+1}=\hat{B}_{K+1}=\hat{C}_{K+1}=0$$

$$\hat{A}_1=\hat{C}_1=0$$

$$\hat{B}_1=1$$

The pressure and gradient of pressure for k=2,K at interfaces can be written as

$$\hat{p}_k=\hat{A}_k+\hat{B}_kp_s+\hat{C}_k\left(\frac{T_{vk-1}+T_{vk}}{T_{0k-1}+T_{0k}}\right)^{C_p/R_d}$$

$$\frac{\partial\hat{p}_k}{a\partial s}=\hat{B}_k\frac{\partial p_s}{a\partial s}+\frac{\hat{C}_k}{T_{vk-1}+T_{vk}}\frac{C_p}{R_d}\left(\frac{T_{vk-1}+T_{vk}}{T_{0k-1}+T_{0k}}\right)^{C_p/R_d}\left(\frac{\partial T_{vk-1}}{a\partial s}+\frac{\partial T_{vk}}{a\partial s}\right)$$

where s can be either latitude and longitude, and all of them are zero when k=K+1. So the pressure equation in the previous section for all levels is reduced to have only the surface pressure equation. It provides a computational saving for both integration time and memory space, though the pressure gradient in momentum equation will have to include the temperature gradient, thus it make a slight complication in dynamical code, especially the semi-implicit scheme.

4. Parallel runs and their performances

Three parallel runs with sigma, sigma-pressure and sigma-theta generalized nybrid GFS have been conducted during October to December 2005. The resolutions of these parallel runs are the same as operational GFS with T382 and 64 vertical layers. Since hybrid-theta performed the best among these three parallel runs during the period, it kept running as parallel until now. Highlights of these runs will be presented here.

Figure 1 shows the anomaly correlation of 500 hPa height after 5-day integration for a period from October to December 2005from the results of NCEP operational GFS (EXPo), generalized-coordinate in sigma (EXPs), in sigma-pressure (EXPp), and in sigma-theta (EXPt). It indicates that generalized-coordinated runs have about the same score as the operational NCEP GFS.

Figure 2 show the comparison of tropical wind root-mean-square errors on 200 hPa among three parallel runs; sigma (EXPs), sigma-pressure (EXPp) and sigma-theta (EXPt), and operational GFS (EXPo). It clearly shows that hybrid-theta has better score than other two parallel runs and operational GFS. Since sigma-theta may provide relatively accurate vertical motion, it shows consistently better scores over all layers along tropical area that others.

9. Discussion

A detailed description of the equation set with finite difference scheme in vertical was provided in the last extended abstract in Juang 2005. To have a complete set, we provide all discretized equations, which have been given in the last extended abstract. Here, we present results from several parallel runs with different vertical hybrid coordinates.

In general, the statistical verification scores from the generalized hybrid GFS are similar to the operational GFS in different vertical hybrid configurations, in terms of height anomaly correlation scores for standard layers, such as 1000 hPa and 500 hPa. Nevertheless, the wind scores over tropic area, the generalized hybrid-theta coordinates performs better than sigma and sigma-pressure hybrid coordinates, even better than operational GFS.

The statistical scores for the past hurricane season, year 2005, the hybrid-theta out-performed among operational GFS and two ready-to operational parallel versions of GFS. Thus, the generalized hybrid GFS is preparing for possible next implementation of NCEP operational GFS.

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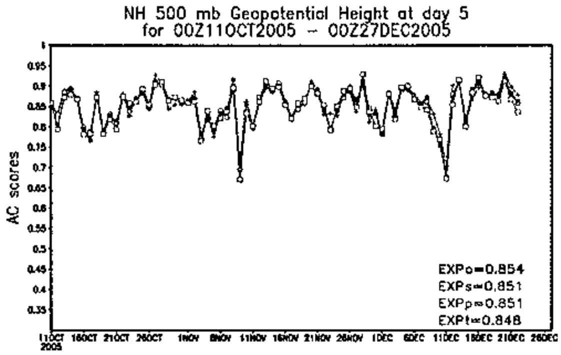

Fig. 1 Anomaly correlations of operational GFS (EXPo), generalized-coordinate sigma (EXPs), sigma-pressure (EXPp), and sigma-theta (EXPt).

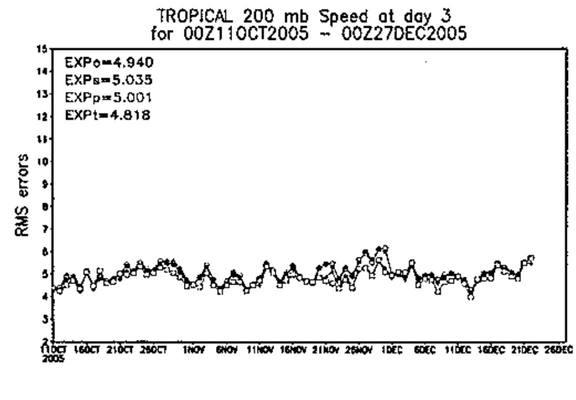

Fig. 2 Root-mean-square error of tropical wins speed on 200 hPa after 72 h forecast for sigma, sigma-pressure and sigma-theta vertical coordinates of NCEP generalized hybrid GFS.