Power-Law Scaling of Tropical Cyclone Precipitation

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1. Introduction

Occurring every season, tropical cyclone (TC) is one of the fiercest and most devastating weather phenomena. Although progress has been made to understand and forecast this amazing feature using advanced observation platforms, numerical models and analytic theories, many important features of TC are still emerging after analyzing various data observationally, numerically and analytically. In this study, we present a unique structure, the spatial spectrum with power-law scaling, from radar observations. The scaling not only stirs our curiosity for interpretation but also provides another simple but dynamic measure for model verifications. The rain-rate distributions derived from reflectivity are decomposed by the two-dimensional Fast Fourier transform (FFT), then the amplitudes in the two-dimensional wavenumber space are averaged at each horizontal wavenumber to form the one-dimensional spectrum at each hour. Observations of two tropical cyclones, one over western Atlantic and the other over western Pacific are analyzed, and results are presented.

2. Hurricane Jeanne over Western Atlantic

During the period between 26 and 28 September, 2004, Hurricane Jeanne made landfall on the east coast of the Florida peninsula, moved northwestward, and took a turn over Georgia toward northeastward approaching New England. Figure 1 illustrates the averaged rainrate over the 72-hour period, and the track is clearly shown. The rainrates are converted from the observed reflectivity through $Z = 300\ R^{1.5}$, where Z is reflectivity in dbz and R the rainrate in mm/hr. The original reflectivity has a resolution of less than 2 km.

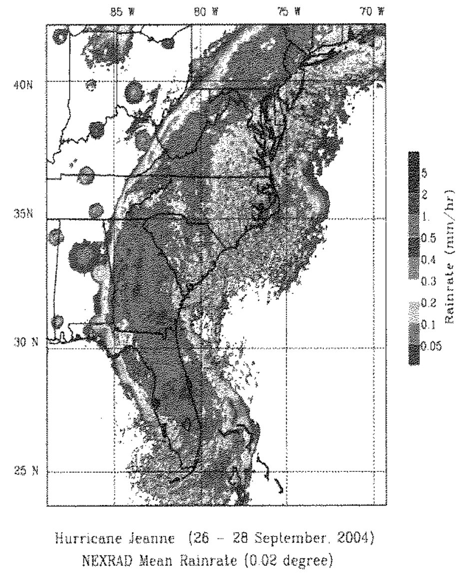

Figure 1. Mean rainrate of Hurricane Jeanne over 26-28 September, 2004.

On each hour during these 3 days, the rainrate distributions derived from the composite NEXRAD observations are spectrally decomposed into two-dimensional (2D) wavenumber space using FFT. These 2D spectra are angularly averaged to produce the one-dimensional (1D) spatial spectra. The hourly spectra are plotted in the log-log frame as shown in Fig. 2(a), (b), and (c) for 26, 27, and 28 September, respectively. Each panel contains 24 hourly spectra. In addition, the dark black dots are their respective daily averages. Among these spectra, one prominent feature is the constant slope over the high wavenumber range between 90 and 350 with an up-turn beyond 350 up to the high-end resolution of 512. The up-turn is the consequence of the spectral aliasing (Kaimal and Finnigan, 1994; Skamarock, 2004). Based on these hourly spectra, the 72-hour average spectrum is plotted in Fig. 2(d) along with the daily averages. At least from these averages, the spectra illustrate a common feature: the -4/3 exponent over the spectral range of wavenumbers higher than 90. They also appear to show additional exponents over two intermediate (30 – 90, and 10 – 30) and low (4 – 10) wavenumber ranges.

To further quantify the power-law exponents on these spectra, a linear least-square fit procedure is employed at 4 different wavenumber ranges. Over the wavenumber range between 90 and 350, the average exponent is -1.33 ± 0.013. The time series of the hourly exponents are presented in Fig. 3. For the higher intermediate range (30 – 90), the average exponent is -0.65 ± 0.047. For these two ranges, the slopes representing the exponents in the log-log plots are quite constant, because the standard deviations resulting from temporal averaging are at least an order of magnitude less than their respective averages. Furthermore, even for each hour, the exponents over these two wavenumber ranges are quite constant because of the smallness of their respective standard deviations.

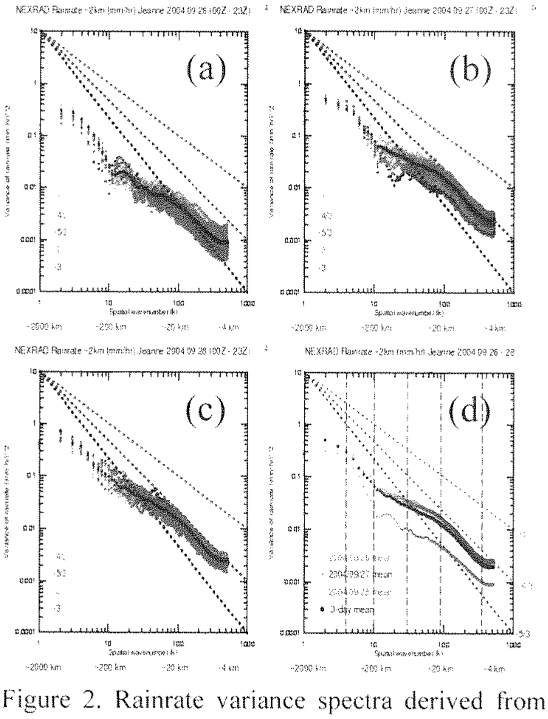

Figure 2. Rainrate variance spectra derived from rainfall patterns of Hurricane Jeanne. Hourly spectra of 26, 27 and 28 September, 2004 are shown in (a), (b) and (c), respectively. The black dots are for daily averages. The 72-hour average spectrum in black dots is shown in (d), in addition to the daily averages. The vertical dashed lines separate 4 wavenumber ranges.

For the two lower wavenumber ranges (10 – 30, and 4 – 10), the exponents are -0.74 ± 0.13 and -1.81 ± 0.19, respectively. But their variability over 3 days is large. Although these exponents look like linear in the 72-hour average spectrum (Fig. 2(d)), their standard deviations are quite larges, as indicated in the time series (Fig. 3). These multiple exponents appear in different wavenumber ranges suggest the possibility of multifractal structure of these hurricane rainfall patterns. Additional statistical tests are required to process these data sets such that multifractals can be demonstrated.

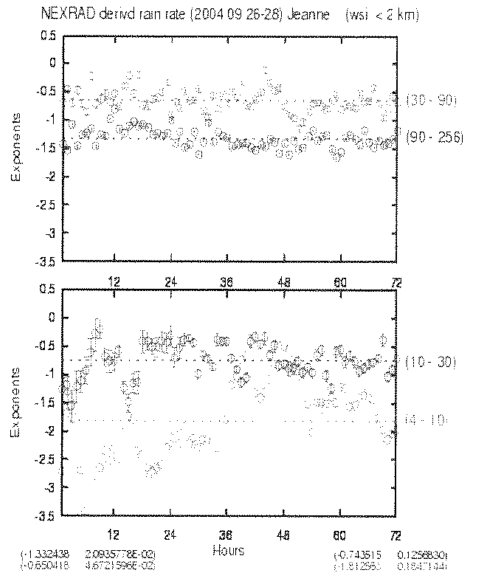

Figure 3. Time series of spectral exponents over wavenumber bands over 72 hours for Hurricane Jeanne. Upper panel: (90 – 350) and (30 – 90). Lower panel: (10 – 30) and (4– 10).

3. Typhoon Aere over Western Pacific

Typhoon Aere took an unusual track passing the Island of Taiwan from northeast to west-southwest during 23-25 August, 2004. Figure 4 depicts the average rainrate over this period based on the composite reflectivity observed by 4 weather radars operated by the Central Weather Bureau at Taipei. The Z-R relationship is $Z = 32.5\ R^{1.65}$ for converting the composite reflectivity to rainrate. The horizontal resolution is 0.0125°.

As being treated for Hurricane Jeanne, the horizontal rainrate distributions are decomposed by 2D FFT, and the resulted 2D wavenumber distributions are angularly averaged to form the 1D spectrum for each hour of these 3 days. Figures 5(a), 5(b) and 5(c) display the rainrate spectra for 23, 24 and 25 August, respectively. In each panel, there are 24 spectra for each hour in different symbols and colors, in addition to the daily average spectrum in black dots. The spectra are steeper than those of Hurricane Jeanne for the wavenumber higher than 10 in the exponent range between -4/3 and -2. The daily and 72-hour average spectra are plotted in Fig. 5(d), and look similar. However, there are delicate differences among various wavenumber ranges.

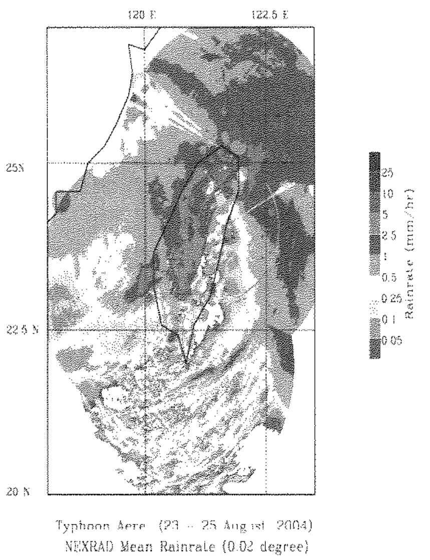

Figure 4. Mean rainrate of Typhoon Aere over 23-25 August, 2004.

As done with Hurricane Jeanne, the exponents of each hourly spectrum are estimated by the best linear fit procedure at different wavenumber ranges. The time series of exponents are shown in Fig. 6.

The exponents for the two higher wavenumber ranges are different, but they are close as seen in the upper panel: -1.46±0.024 (90 – 350) and -1.77±0.064 (30 – 90). For these exponents the best fits are quite linear with very small error ranges. On the other hand, the error ranges are larger for the best linear-fits in the lower wavenumber ranges: -1.49±0.13 (10 – 30) and -0.94±0.24 (4 – 10).

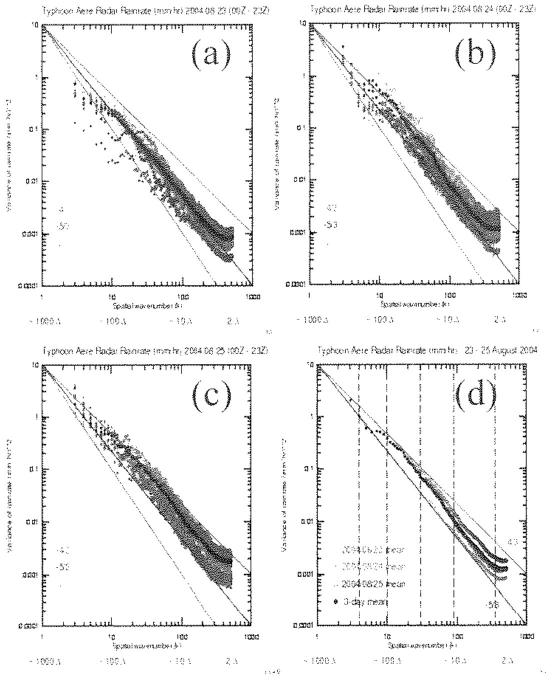

Figure 5. Rainrate variance spectra derived from rainfall patterns of Typhoon Aere. Hourly spectra of 23, 24 and 25 August, 2004 are shown in (a), (b) and (c), respectively. The black dots are for daily averages. The 72-hour average spectrum in black dots is shown in (d), in addition to the daily averages. The vertical dashed lines separate 4 wavenumber bands for further analyses.

In general, these exponents among different wavenumber ranges are much closer than those found in the Hurricane Jeanne spectra. Therefore, the possible structure of multifractal seems less apparent, though not impossible. Again, further tests using other statistical methods are warranted.

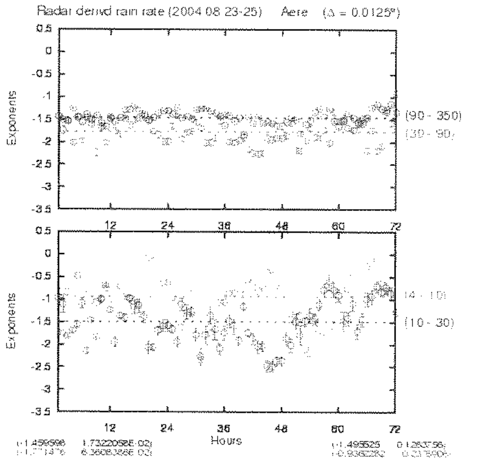

Figure 6. Time series of spectral exponents over wavenumber bands over 72 hours for Typhoon Aere. Upper panel: (90 – 350) and (30 – 90). Lower panel: (10 – 30) and (4– 10).

4. Summary

In this investigation of tropical-cyclone precipitation, a different perspective of rainrate distributions is presented: multiple power-law scaling in the wavenumber space. Although complex and abstract, a new paradigm for TC precipitation is introduced. Furthermore, such methodology of spectral representation is suggested for a simple but dynamic measure for verifications of rainfall between model forecasts and observations.

References

Kaimal, J.C., and J.J. Finnigan, 1994: *Atmospheric Boundary Layer Flows*. Oxford University Press, 289 pp.

Skamarock, W.C., 2004: Evaluating mesoscale NWP models using kinetic energy spectra. *Mon. Wea. Rev.*, **132**, 3019-3032