THE IMPLMENTATION OF HYBRID VERTICAL COORDINATES TO NCEP GFS

Hann-Ming Henry Juang
Environmental Modeling Center, NCEP, Washington DC
Henry.Juang@noaa.gov

z1. Introduction

The generalized vertical hybrid coordinates for atmospheric modeling has been developed (Simmons and Burridge 1981; Zhu et al 1992; Konor and Arakawa1997; Johnson and Yuan 1998; Benjamin et al 2004). With generalized hybrid vertical coordinates, the atmospheric model can be integrated along any different types of coordinate surfaces. The coordinates near surface and lower atmosphere used to apply terrain following sigma coordinates, but over the upper atmosphere, they had better to compute on quasi-horizontal such as pressure surfaces or isentropic surfaces to reduce the numerical errors due to mis-estimated vertical motions. The combination of these coordinates as hybrid coordinates can take advantage of individual type of the coordinate surfaces for numerical purpose. The dynamics group in EMC has made an effort to move into this direction as well.

We plan to have our own approach to generalized vertical hybrid coordinates with an incremental implementation. For example, all prognostic variables as what we used are included as the prognostic variables in the hybrid vertical coordinates system. After the selection of prognostic variables, we keep using spectral computation in horizontal and finite difference in vertical for the first implementation, though we plan to have semi-Lagrangian, finite or spectral element in vertical. Due to the hybrid coordinate equation set is different from what we have, new discretization in vertical is required to satisfy energy and angular momentum conservations. The matrixes used for semi-implicit time integration have to be modified due to different vertical discretization in hybrid coordinates.

This report describes a short summary of current result of a discretization of a hydrostatic version of a primitive equation global model on spherical and generalized hybrid coordinates. The detailed can be found in NCEP office note number 445 in http://www.emc.ncep.noaa.gov/officnotes. We will still keep spectral computation in horizontal without mentioning, and use finite difference in vertical. For backward compatible, we will use virtual temperature as model prognostic variable. Section 2 lists the completed set of all continuous equations, and introduces a map factor to rewrite equation set on spherical coordinates to regular latitude/longitude pseudo spherical coordinate. Section 3 discusses the vertical constraints in continuous forms ready for detailed discretization in section 4. Section 5 illustrated the process to solve vertical flux for vertical advections. Section 6 describes the semi-implicit method with the help of linearized equations of divergence, virtual temperature and pressure. A specific definition of hybrid coordinate is introduced in section 7, some results and comparison in section 8, and discussion is in section 9.

2. Hydrostatic system on spherical and generalized hybrid coordinates

The primitive hydrostatic system on spherical coordinates in horizontal and generalized hybrid coordinate in vertical can be obtained from a text book, such as Haltiner and Williams (1979), and it can be written as

$$\frac{\partial u^*}{\partial t} = -m^2 u^* \frac{\partial u^*}{a\partial\lambda} - m^2 v^* \frac{\partial u^*}{a\partial\varphi} - \dot{\zeta}\frac{\partial u^*}{\partial\zeta} - R_d \frac{T_v}{p}\frac{\partial p}{a\partial\lambda} - g\frac{\partial z}{a\partial\lambda} + f_s v^* + F_u^*$$

$$\frac{\partial v^*}{\partial t} = -m^2 u^* \frac{\partial v^*}{a\partial\lambda} - m^2 v^* \frac{\partial v^*}{a\partial\varphi} - \dot{\zeta}\frac{\partial v^*}{\partial\zeta} - R_d \frac{T_v}{p}\frac{\partial p}{a\partial\varphi} - g\frac{\partial z}{a\partial\varphi} - f_s u^*$$

$$-m^2 \frac{s^{*2}}{a}\sin\phi + F_v^*$$

$$\frac{\partial T_v}{\partial t} = -m^2 u^* \frac{\partial T_v}{a\partial\lambda} - m^2 v^* \frac{\partial T_v}{a\partial\varphi} - \dot{\zeta}\frac{\partial T_v}{\partial\zeta} + \kappa\frac{T_v}{p}\frac{dp}{dt} + F_{T_m}$$

$$\frac{\partial(\partial p/\partial\zeta)}{\partial t} = -m^2\frac{\partial u^*(\partial p/\partial\zeta)}{a\partial\lambda} - m^2\frac{\partial v^*(\partial p/\partial\zeta)}{a\partial\varphi} - \frac{\partial\dot{\zeta}(\partial p/\partial\zeta)}{\partial\zeta}$$

$$\frac{\partial q_i}{\partial t} = -m^2 u^* \frac{\partial q_i}{a\partial\lambda} - m^2 v^* \frac{\partial q_i}{a\partial\varphi} - \dot{\zeta}\frac{\partial q_i}{\partial\zeta} + F_{q_i}$$

where

$$m = \frac{1}{\cos\phi}$$

$$\Delta\varphi = m\Delta\phi = \frac{1}{\cos\phi}\Delta\phi$$

In this case, true spherical λ and ϕ, longitude and latitude, coordinates are mapped into pseudo-spherical coordinates by λ and φ. Also, where u and v are horizontal winds, $\dot{\zeta}$ is vertical coordinate velocity, Tv is virtual temperature, p is pressure, q with index i are tracers including specific humidity. F with related suffix as the last term in each equation are parameterization of model physics. The horizontal coordinates λ and ϕ are spherical longitude and latitude. Others are traditional used, such as g is gravitational force, a is earth radius, fs is sine component of Coriolis force, z is height from surface.Note that, this equation set here is a close system to solve, except hybrid vertical velocity, which will de discussed in section 5 later.

3. Constraints for Vertical Discretization

In this section, we summarize continuous form of the constraints proposed by Arakawa and Lamb (1977) in designing a vertical difference scheme. Three constraints are following: