The Formation Mechanism of the Concentric Eyewalls

Jia-Yi Peng and Ben Jong-Dao Jou

Department of Atmospheric Sciences, National Taiwan University,Taipei 106

1. ABSTRACT

The vortex will mutually adjust the thermodynamic field and dynamic field to a state of gradient balance whilst forced by a cold source, namely, the gradient adjustment process, which is dealt with a linearized two-layer axisymmetric model in this paper. The analyses show that on account of inhermogeneous distribution of radial cold source, the adjustment of the thermodynamic and dynamic fields results in the double-peak tangential wind feature, which is analogous to the concentric eyewalls in strong tropical cyclone. Consequently, the gradient adjustment may be another possible mechanism of the formation of the concentric double eyewalls tropical cyclone.

Key words: tropical cyclone, concentric eyewalls, gradient adjustment

2. INTRODUCTION

The concentric eyewalls structure often displays in the strong tropical cyclone which is defined as the minimum sea level pressure is below 970hPa with maximum tangential wind speed more than 45m/s (Chen, 1987). For example, the hurricane ilbert Willoughby et al. (1989) estimated its minimum sea level pressure of 888hPa, with the inner eyewall in the radius of 8-20km and the outer eyewall 55-100km. The more detailed analysis of ilbert (Black and Willoughby, 1992) showed that the primary eyewall appeared firstly, and the outer eyewall displayed remarkably in the decaying period of the hurricane, later on, the outer eyewall strengthened and contracted while the inner eyewall showed sign of weakening. Finally, the outer eyewall replaced the inner eyewall and completed the eyewall replacement cycle (refer to Fig.1).

Indeed, the mechanism by which concentric eyewalls initially form is not clearly. Willoughby et al. (1984,1988) speculated that symmetric instability plays an important role in the formation of the outer eyewall. However, how to explain the emergence of symmetric instability in the upper troposphere, and the authors could not illustrate the location of the outer eyewall. On the other hand, Montgomery and Kallenbach (1997) suggested that the concentric eyewalls might be the result of radial propagating linear vortex Rossby waves and the presence of a critical radius in the tropical cyclone. However, the propagation of the vortex Rossby wave bear a close relation to the radial vorticity gradient, then the vortex Rossby wave will be confined in the radius of the maximum winds. Recently, Nong and Emanuel (2003) have discussed the formation of the concentric eyewalls with the aid of axisymmetric model. Their simulations show that the secondary eyewall may result from a finite-amplitude WISHE instability, triggered by external forcing.

In fact, it is evident from Fig.1 that the outer eyewall displayed noticeably in the weakening period of the tropical cyclone, which may be associated with the decreasing heat due to the effects of cumulus clouds. Shapiro and Willoughby (1982) and Willoughby et al. (1982) designed a symmetric model (SW model) to diagnose the secondary circulation induced by point source of heat in balanced, axisymmetric vortices. For a heat source near the radius of the maximum tangential wind, a sharp peak of the tangential wind tendency lay inside the maximum of the wind itself, so the maximum propagated inward in response to heating, which provided a plausible physical explanation for the contraction of the outer wind maximum. Nonetheless, the SW model could not simulate the double-peak structure of the tangential wind that often displayed in concentric eyewalls typhoon.

Schubert et al.(1980) discussed geostrophic adjustment in an axisymmetric vortex with the aid of a linearized two-layer model. The model governs small amplitude, forced, axisymmetric perturbations on a basic-state tangential flow that is independent of height. When the basic flow is at rest, solutions for the transient and final adjusted state are found by the method of Hankel transformations. For the non-resting basic state, they only discuss the final adjusted state without considering the heat source. This paper will further discuss the transient response of a very strong basic state vortex to a cold source, which induced the weakening of the strong tropical cyclone. It is shown in this study that gradient adjustment process may be another formation mechanism of the concentric eyewalls.

The basic outline of this paper is as follows. After deriving the governing equations for small-amplitude perturbations and the method for the solution of the equation in section 2, we present the numerical results concerning the formation of concentric eyewalls in the decaying phase of strong tropical cyclone in section 3, and Section 4 discusses the contraction of the outer eyewall, a summary and a discussion for future work are given in Section 5.

3. GOVERNING EQUATIONS

The response of an axisymmetric vortex to the cold source $Q\ (r,t)$ is discussed in this paper. By using cylindrical coordinates in the horizontal and pressure in the vertical, the axisymmetric form of the primitive equations can be written as follows:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+\omega\frac{\partial u}{\partial p}-(f+\frac{v}{r})v+\frac{\partial \phi}{\partial r}=0, \quad (1a)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial r}+\omega\frac{\partial v}{\partial p}+(f+\frac{v}{r})u=0, \quad (1b)$$

$$\frac{\partial ru}{r\partial r}+\frac{\partial \omega}{\partial p}=0, \quad (1c)$$

$$\frac{\partial}{\partial t}\left(-\frac{\partial\phi}{\partial p}\right)+u\frac{\partial}{\partial r}\left(-\frac{\partial\phi}{\partial p}\right)-\sigma\omega=\frac{R}{C_p p}Q(r,t), \quad (1d)$$

where u is the radial component of velocity, v the tangential component, ω the vertical P velocity ($\omega=dp/dt$), ϕ the geopotential, f the constant Coriolis parameter (here, $f=5\times10^{-5}$(1/s)), and σ the static stability defined by $\sigma=-\rho^{-1}(\partial\ln\theta/\partial p)$.

In order to simplify the vertical structure of this system as much as possible, we follow the geostrophic adjustment study of Schubert et al.(1980) by considering the standard two-layer model version of (1a)-(1d). In addition, we restrict our attention to small perturbations about a basic state of gradient balance, i.e., $\bar{u}=\bar{\omega}=0$ and $(f+\bar{v}/r)\bar{v}=\partial\bar{\phi}/\partial r$ with $\bar{v}$ assumed to be a function of r but not of p. With (1a)-(1c) applied at level 1 (250hPa) and level 3 (750hPa), and (1d) applied at level 2 (500hPa), and requiring that $\omega=0$ at the top (0hPa) and bottom pressure surfaces (1000hPa). (1a)-(1d) reduce to the next six equations:

$$\frac{\partial u_d}{\partial t}-(f+\frac{2\bar{v}}{r})v_d+\frac{\partial\phi_d}{\partial r}=0, \quad (2a)$$

$$\frac{\partial v_d}{\partial t}+(f+\frac{\partial r\bar{v}}{r\partial r})u_d=0, \quad (2b)$$

$$\frac{\partial\phi_d}{\partial t}+\frac{\bar{\sigma}_2(\Delta p)^2}{2}\frac{\partial ru_d}{r\partial r}=\frac{R}{C_p}Q_2(r,t), \quad (2c)$$

$$\frac{\partial u_a}{\partial t}-(f+\frac{2\bar{v}}{r})v_a+\frac{\partial\phi_a}{\partial r}=0, \quad (2d)$$

$$\frac{\partial v_a}{\partial t}+(f+\frac{\partial r\bar{v}}{r\partial r})u_a=0, \quad (2e)$$

$$\frac{\partial ru_a}{r\partial r}=0, \quad (2f)$$

in which $\Delta p=500hPa$, and where the subscript a refers to the addition between the upper and lower levels, i.e., $\phi_a=\phi_1+\phi_3$, $u_a=u_1+u_3$, $v_a=v_1+v_3$; and the subscript d presents the difference between the upper and lower levels, i.e., $\phi_d=\phi_1-\phi_3$, $u_d=u_1-u_3$, $v_d=v_1-v_3$. Specifically, ϕ_d displays the depth of the two levels and u_d indicates the shear of wind between the upper and lower levels.

It is convenient to convert (2a)-(2f) into non-dimensional form by choosing units of time, horizontal distance and velocity as $1/f=5.56$ hr, $c/f=800$ km and $c=40$ m/s. Here $c^2=\frac{1}{2}\bar{\sigma}_2(\Delta p)^2$ is the square of the phase speed of a pure internal gravity wave. Then the dimensionless quantities: time, radial distance, basic state tangential wind, shear velocity components, thickness, the additions of radial wind, tangential wind and geopotential height, source term become ft, $(f/c)r$, $\bar{v}/c$, u_d/c, v_d/c, ϕ_d/c^2, u_a/c, v_a/c, ϕ_a/c^2, $(R/C_p)(Q_2/c^2)$, respectively. In short, using the symbols $t, r, \bar{v}, u_d, v_d, \phi_d, u_a, v_a, \phi_a$ and $Q(r,t)$ for these new dimensionless variables, (2a)-(2f) become:

$$\frac{\partial u_d}{\partial t}-(1+\frac{2\bar{v}}{r})v_d+\frac{\partial\phi_d}{\partial r}=0, \quad (3a)$$

$$\frac{\partial v_d}{\partial t}+(1+\bar{\zeta})u_d=0, \quad (3b)$$

$$\frac{\partial\phi_d}{\partial t}+\frac{\partial ru_d}{r\partial r}=Q(r,t), \quad (3c)$$

$$\frac{\partial u_a}{\partial t} - (1+\frac{2\bar{v}}{r})v_a + \frac{\partial \phi_a}{\partial r} = 0, \quad (3d)$$

$$\frac{\partial v_a}{\partial t} + (1+\bar{\zeta})u_a = 0, \quad (3e)$$

$$\frac{\partial r u_a}{r\partial r} = 0, \quad (3f)$$

where $\bar{\zeta} = \frac{\partial r\bar{v}}{r\partial r} = \frac{\bar{v}}{r} + \frac{\partial \bar{v}}{\partial r}$ is the basic state relative vorticity. From equation (3f) we will know that ru_a must be independent of r and only a constant, considering the boundary condition: $r \to 0$, $u_a \to 0$,which will induce to:

$$u_a = 0, \quad (4)$$

by substituting (4) into (3e), we obtain that v_a does not change with time, and recalling that $v_a|_{t=0} = 0$, we may write:

$$v_a = 0. \quad (5)$$

From (4) and (5) we will indicate that the barotropic component of perturbation wind is always zero, then substituting (4) and (5) into (3d) gives that ϕ_a is also independent of r, followed by use of $\phi_a|_{r\to\infty} = 0$, leads to:

$$\phi_a = 0. \quad (6)$$

The equations (3a)-(3c) is identical with that of (2.8)-(2.10) in the paper of Schubert et al. (1980). The solutions of the forgoing mentioned equations are discussed in order to study the response of the vortex on the heat forcing.

4. FORMATION OF CONCENTRIC EYEWALLS

In order to bogus the initial balance vortex similar to a actual tropical cyclone, we define the basic state tangential wind as follows:

$$\bar{v} = a[2 - e^{-br}(b^2r^2 + 2br + 2)]/(br),$$

the corresponding relative vorticity in the form of $\bar{\zeta} = ab^2re^{-br}$, where $a = 4.0$, $b = 40.0$. Figure 2 portrays the radial profiles of tangential wind and relative vorticity of the basic state. It is evident from Fig.2a that for the initial balance vortex, the tangential wind is zero at the center, and increase with r up to the maximum (the dimensional value, $V_{max} = 62.4$ m/s located at $r_{max} = 67.2$ km) then decrease with r, which is analogous to a actual tropical cyclone. Furthermore, we include a specified rate of external cold source term in (3c) that can be factored into space-dependent and time-dependent parts, namely,

$$Q(r,t) = Q_r(r)Q_t(t),$$

where $Q_t(t) = \alpha^2te^{-\alpha t}$, small α corresponds to slow forcing and large α to rapid forcing, but the total forcing is independent of α (here, $\alpha = 1$) since $\int_0^\infty \alpha^2te^{-\alpha t}dt = 1$. The spatial distribution of the cold source is given in the form of $Q_r(r) = C*\{erf[A(r-B)]-1\}$, in which $C = 4.0$, and $A = 40.0$ displays the spatial variation intensity of the cold source, and parameter B indicates the location of the cold source. In this section, three experiments (TEST1, TEST2, TEST3) are developed by choosing the values of B, i.e., B=0.14; B=0.10; B=0.17, respectively. In fact, the corresponding dimensional value of the strongest cold source is about -2.97K/hr. Fig.3, Fig.5a and Fig.6a give the time- and spatial-variation of the cold source. It is clear that the cold source increase from zero and up to the maximum at $t=1.0$, from then on decreasing to zero till $t=10.0$ (Fig.3a). On the other hand, the maximum intensity of the cold source is situated in the center of the vortex, and decrease with r quickly (see Fig.3b, Fig.5a and Fig.6a). The forgoing mentioned cold source is selected to consider the weakening of the cumulus convection, which is attributed to the ocean-atmosphere interactions in cold SST regions formed in the trail of the tropical cyclone event, such as tropical cyclone Kai-Tak in 2000 (Lin et al.2003). Anomalously cold SST patches around 100-km spatial scale up to 6^0C below the surrounding warm tropical ocean SST are found along the trail of tropical cyclone tracks as cold, deep waters are entrained up to the mixed layer due to tropical cyclone forcing. On the contrary, the local remarkable low SST will change the surface fluxes and further modify the structure, intensity and track of the corresponding tropical cyclone.

According to the foregoing experiment design, we can calculate the vertical shear of the tangential wind perturbations (v_d) in different time steps, and obtain the vertical shear of the radial wind perturbations (u_d) and thickness (ϕ_d). Specifically, here we only consider the adjustment process in level 3. Figures 4a and 4b depict the time variation of the radial wind perturbations in level 3 of TEST1. It can be seen from these figures that on account of the cold source, the initial balance vortex displays unbalance and begin to adjust between wind and geopotential fields. The radial wind perturbations (u_3) located around vortex center increase gradually, then accompanied with the outward propagating of the gravity-inertia wave, u_3 begin to decrease gradually. Nevertheless, after the gravity-inertia wave propagates outward, the vortex arrives at a new balance with u_3 recovered to zero. In addition, we also find from Figure 4a and 4b that there exists a radial wind maximum area that is situated at $r=0.15$, i.e., the location of the noticeable spatial variation of the cold source. In other words, the spatial distribution of the cold source have remarkable effect on the radial profile of radial wind perturbations, which can be also found from (3a) and (3c).

Fig.4c displays the evolution of the tangential wind perturbations (v_3) in the adjustment process. Unlike the u_3, the v_3 is always decreasing with time till up to the steady state. Likewise, since the evolution of v_3 is not only associated with u_3 but also with the relative vorticity of the initial balance vortex, the maximum of v_3 is located in the proximity of $r=0.1$, which do not overlap with that of u_3.

Fig.4d delineates the time variation of the total tangential wind ($V_3=\overline{v}+v_3$) at level 3. By referring to Fig.4d, we can see that the vortex begins to adjust corresponding with the variation of the tangential wind distribution after it is forced by the cold source. On account of the cold source, the tangential wind speeds decrease gradually with different modification at each location. Consequently, the radial profile of the tangential wind gradually displays double-peak structure. In the beginning, the inner peak wind speed is larger than the outer one. Then the difference of double peaks gradually decreases, and finally the outer peak is larger than the inner one. The evolution of the tangential wind is analogous to that of hurricane ilbert (see Fig.1). The foregoing discussion illustrates that under the condition of the cold source forcing, the gradient balance vortex becomes unbalanced. In the adjustment process, the tangential wind profile will be similar to that of the concentric double-eyewall structure. Therefore, the gradient adjustment process may be used to illustrate the formation of the concentric eyewalls in a tropical cyclone.

Fig.5 and Fig.6 portray the simulations of TEST2 and TEST3. The time variation of the radial wind (see Fig.5b and 6b) and tangential wind (see Fig.5c and 6c) perturbations at level 3 of TEST2 are similar to that of TEST3. The comparison of the two tests indicates that the location of the maximum radial wind perturbations agree roughly with that of the remarkable radial variation of the cold source, which demonstrates that the evolution of the radial wind perturbation is closely related with the cold source. In comparison, the maximum perturbation tangential wind is not located at the position of the noticeable radial variation of the cold source, which is attributed to the structure of the initial balance vortex. As such, the initial balance vortex structure has remarkable effect on the evolution of tangential perturbations in the adjustment process. As is clear from figure 5d and 6d, the tangential wind at level 3 is always decreasing with different value in different location, some time later, the radial distributions of the tangential wind in two tests also present double-peak structure. The difference of TEST1, TEST2 and TEST3 is that the different positions of the maximum perturbation tangential wind is owing to the different radial distribution of the cold source, resulting in different double-peak structure in three tests. Specifically, in the prior period of the formation of the double-peak structure for TEST1, the tangential wind of the inner peak is larger than that of the outer one, and vice versa in the late phase. For TEST2, the tangential wind of the inner peak decrease sharply with the velocity smaller than that of the outer peak in the total period. The feature of TEST3 is in contradiction to that of TEST2. As a result, forced by different structure of the cold source, the initial balance vortex will probably change into double-peak structure that is analogous to that of concentric eyewalls tropical cyclone.

5. DISCUSSION AND CONCLUSIONS

In the context of a linearized two-layer axisymmetric model, the gradient adjustment process of the initial balance vortex forced by a heat source is discussed in this paper. It turns out that owing to the forcing of the cold source, in the decaying period of the strong tropical cyclone, its tangential wind will display double-peak distribution analogous to that of concentric eyewalls tropical cyclone. Therefore, the gradient adjustment process of the initial balance vortex forced by a cold source may be the mechanism of the formation of the concentric eyewalls. Nevertheless, the dynamic analyses in this study are based on a linearized dry model, where the interaction of the cloud microphysical process and the dynamic field is not considered. However, the moisture process corresponding with the strong convection plays an important role in the formation of the concentric eyewalls. So the linearized two-layer model used in this paper is very simple and ideal, the more complex model and analysis will be presented in the future work.

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Figures(omitted)