Applying Horizontal Diffusion on Pressure Surface to Mesoscale Models on Terrain-Following Coordinates

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1. Introduction

It is well known that horizontal diffusion over mountain areas should be used on horizontal surfaces, otherwise an erroneous vertical mixing of different horizontal locations through the horizontal diffusion on the terrain-following coordinate surfaces will occur. The error produced by this erroneous vertical mixing could be dramatically large for the variable of concerns that has stratification in vertical, such as temperature and humidity. In sigma-coordinate models, the straight way to avoid this kind of errors is to apply horizontal diffusion on the real horizontal (Zängl 2002) or quasi-horizontal pressure surfaces (Moorthi 1997; Kanamitsu 2002).

Horizontal diffusion on the 'real' horizontal in the terrain-following coordinates requires either vertical interpolation to have necessary values on the same height or coordinate transforms to have horizontal diffusion on real- or quasi-horizontal surfaces.. Nevertheless, there is another alternative way for horizontal diffusion on terrain-following coordinates by applying it on perturbation instead of full field under the condition that the distribution of the perturbation is not systematic or the stratification is eliminated in the perturbation.

Perturbation used for horizontal diffusion on terrain-following coordinate will be described in section 2. A full-field horizontal diffusion on pressure surfaces is introduced in section 3. In section 4, three cases are used to illustrate the evidences of problematic perturbation diffusion on terrain-following surfaces in high resolution. The discussion and conclusion are given in section 5.

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2. The perturbation horizontal diffusion on sigma coordinates

The National Centers for Environmental Prediction (NCEP) regional spectral model (RSM) and mesoscale spectral model (MSM) apply fourth-order horizontal diffusion on sigma surface to the perturbation in spectral space (Juang and Kanamitsu 1994, Juang et al. 1997, Juang 2000, Juang and Hong 2001). The perturbation equation with horizontal diffusion can be re-illustrated here as the following

$$\frac{\partial A'}{\partial t}=\frac{\partial A_R^*}{\partial t}-\frac{\partial A_G}{\partial t}-\kappa\nabla_\sigma^4 A' \qquad (2.1)$$

where A can be any prognostic variable, and κ is a constant diffusion coefficient. The first term in the RHS (right-hand-side) is the total forcing computed in the regional grid without horizontal diffusion, so we put * there, the second term in the RHS is the total global tendency interpolated at regional grid, and the third term in the RHS is the horizontal diffusion on sigma surface for perturbation forcing, under the assumption that the horizontal diffusion of base field has been done in second term of RHS.

The perturbation horizontal diffusion on coordinate-surfaces in Eq. 2.1 is a linear term and can be computed in spectral space as following

$$-\frac{1}{\tau_\sigma}\left(\frac{m^4+n^4}{M^4+N^4}\right)[A']_m^n \qquad (2.2)$$

where m and n are the wave numbers in x and y directions, respectively, and M and N are the maximal wave numbers in x and y directions, respectively. [ ] represents the variable in spectral space at wave (m,n), and τ_σ is e-folding time as a weighting coefficient to control the effect of the diffusion, i.e. $\kappa = (\Delta x/\pi)^4/\tau_\sigma$. For numerical stability, implicit time scheme is used. It works quite well for coarse resolution while the perturbation over mountain area is small.

The perturbation given in the previous paragraph is defined as the difference between regional model values and global model values on the same sigma surface. In cases of proximity of model terrains between NCEP RSM/MSM and its driven coarse resolution model, say NCEP GSM, the perturbation will be negligibly small initially, then the horizontal diffusion on sigma coordinate surfaces is reasonably well to smooth out the noise generated in the perturbation after nonlinear integration.

There is a numerical problem for horizontal diffusion on sigma surface over the mountain slope areas. Fig. 1 is a schematic plot to illustrate the problem of the perturbation diffusions, the heavy (light) solid curve and heavy (light) dashed curve indicate the heights of regional (global) model terrain and its related sigma surface, respectively. In case of temperature or moisture field, the value on top of the high terrain from regional model is colder or drier than that from the global model because it represents the value at higher altitude, and the valley or sides of the mountain, the temperature is warmer or more moist in the regional model because of lower altitudes. With this pattern of perturbation, the horizontal diffusion of perturbation on sigma surface will reduce the difference; hence warming (moistening) on top of mountain and cooling (drying) over the sides or valley of the mountains, depends on the environmental conditions.

3. Horizontal diffusion on pressure surface in sigma coordinates

In high resolution, the numerical error of horizontal diffusion on sigma surface will appear non-negligibly small due to the larger perturbation differences along the mountain slopes, and eventually it ruins the forecast or simulation. The immediate solution to this is to apply horizontal diffusion on horizontal or quasi-horizontal surfaces, such as constant height or pressure surfaces, instead of modifying the definition of perturbation. And for the predicted variables in NCEP RSM/MSM, only temperature and moisture require modification for their horizontal diffusion due to their stratification while the horizontal wind fields in NCEP RSM/MSM and vertical wind field and non-hydrostatic pressure in NCEP MSM are not always monotonically increasing or decreasing with heights, thus they can be diffused on sigma surface for smoothing purpose.

Since sigma coordinates is related to pressure, thus it is easier to use pressure surface to be the quasi-horizontal surface for horizontal diffusion than to use height. Due to the further complexity in fourth order, second order is used. The second-order diffusion on pressure surface can be written as

$$\kappa\nabla_p^2 A = \kappa\left(\frac{\partial^2 A}{\partial x^2}+\frac{\partial^2 A}{\partial y^2}\right)_p \qquad (3.1)$$

where A can be either temperature or specific humidity, and κ is a constant coefficient. To use it on sigma surface, we apply coordinate transform, then, the final equation we need for second-order diffusion can be written as

$$\begin{aligned}\nabla_P{}^2 A &= \nabla_\sigma{}^2 A - \frac{\partial A}{\partial\ln\sigma}\nabla_\sigma{}^2\ln P_s \\ &-2\frac{\partial}{\partial\ln\sigma}\left(\frac{\partial A}{\partial x}\frac{\partial\ln P_s}{\partial x}+\frac{\partial A}{\partial y}\frac{\partial\ln P_s}{\partial y}\right) \qquad (3.2). \\ &+\frac{\partial^2 A}{\partial\ln\sigma^2}\left[\left(\frac{\partial\ln P_s}{\partial x}\right)^2+\left(\frac{\partial\ln P_s}{\partial y}\right)^2\right]\end{aligned}$$

Due to the nonlinear terms, the explicit time scheme is used in the diffusion, and t-Δt step should be used in case of explicit forcing for stability in central time differencing as

$$\frac{(A)^{t+\Delta t}-(A)^{t-\Delta t}}{2\Delta t} = ...+\frac{\mu}{\tau_P}\nabla_P{}^2(A)^{t-\Delta t} \qquad (3.3)$$

where $\mu=0.125(\Delta x)^2$ without considering map factor for simplicity, and τ_p should be a value in seconds larger than 2 Δt.

4. Case results

From here after in this section, let s-DIFF refer to the original design of perturbation fourth-order horizontal diffusion on sigma surfaces as described in section two, and p-DIFF to the modified version of full-field second-order horizontal diffusion on pressure surfaces as described in section three.

a.   Winter-time rainfall over Taiwan

The wintertime rainfall of December 1996 over Taiwan was selected to be the case study in this subsection. During the December of 1996, the eastern edge of the Asian continental high-pressure center provides a prevailing northeastern wind over Taiwan and its vicinity. The observed rainfall is shown in Fig. 2 with heavy rainfall spreading over the northeastern Taiwan.

RSM is used for this case. Figure 3 shows monthly averaged rainfall with contour interval of 1 mm/day from (a) the experiment with s-DIFF, and (b) the experiment with p-DIFF. Though both experiments result excessive rainfall as compared to observation, the two local maxima of the rainfall are well predicted. And the result from p-DIFF, in Fig. 3(b), improves the rainfall amount by reducing in about half from the experiment in s-DIFF, in Fig. 3(a). Since this is a terrain induced rainfall, the higher the model terrain resolution is, to resolve better the model terrain, the better the rainfall amount and spatial distribution could be as compared to the observation.

b.   Weak trade-wind condition over Island of Oahu

A weak trade wind case over Hawaii was selected to perform daily weather forecast by using NCEP MSM to illustrate the impact of the horizontal diffusion on sigma and pressure surfaces. The NCEP global analysis (T170, 42 layers) is used as the initial condition, 0000 UTC 25 May 2002 (i.e. 1400 HST 24 May 2002), and NCEP global model forecasts are used as the boundary conditions. First, 10-km 28-layers NCEP RSM was performed with updating every time step of global model tendency by interpolating the global model forecasts at 6-hr intervals and output 10-km NCEP RSM results at every 3 hours, then 1.5-km 28-layer NCEP MSM is nested into 10-km NCEP RSM at 3-hr intervals.

In this clement weather condition, land-sea breeze can be expected due to the effect of diurnal cycle. Fig. 4 shows model 10-m wind valid at 2000 HST 24 May 2002, after 6-hr integration, from experiments of (a) s-DIFF and (b) p-DIFF. It clearly indicates that entire domain is under an averaged trade wind of 5 m/s, but the local wind patterns between two experiments are dramatically different, not only over island but also the vicinity of the coastal waters. For a local time at 2000 HST, it is still in the period within the phase of sea breeze, while there is a high-pressure center covered the area provided a normal diurnal cycle of land-sea breezes. Thus, it is quite easy to identify the erroneous flow pattern in Fig. 4(a), which shows a strong land breeze over the windward side. However, after the modification of the horizontal diffusion on pressure surface, the flow shows a reasonable pattern without land breeze at the windward side and a remarkable lee-side wake, as shown in Fig. 4(b).

c.   Summer monsoon rainfall over North America

This is a North America summer monsoon case over Mexico and the southwest states of US, Arizona and New Mexico (AZNM). The reanalysis data (Kalnay et al. 1996), T62 with 28 layers, are used as the initial condition, lateral boundary and base field for NCEP RSM. The resolution of the NCEP RSM is 20 km in horizontal and 28 layers in vertical. The NCEP RSM performed monthly integration from August 1 1990 with updating sea-surface temperature daily. The time step of the model is 120 sec, the diffusion time step is 6000 sec.

It can be proved that p-DIFF is better than s-DIFF by the daily rainfall evolution over AZNM (from –112 to –102 W and from 32 to 36 N) shown in Fig. 5. It shows the daily rainfall during August 1990 over AZNM for observation (solid curve), p-DIFF (dotted curve) and s-DIFF (dashed curve). Except the event on 21 August, model with s-DIFF and p-DIFF picked up other two major events, one at 6 August, the other during 14-15 August. And the major event has amount up to 12 mm/day caught by the experiment with p-DIFF. It indicates that p-DIFF provided better distribution of the humidity (and temperature as well, not shown) to have better rainfall distribution due to the reasonable moisture support.

5. Conclusion and Discussions

A modified horizontal diffusion is implemented into the different version of NCEP RSM and NCEP MSM to replace the original method. Due to the definition of the perturbation in NCEP RSM/MSM, the original scheme becomes non-negligible systematic errors as the horizontal resolution increasing with large deviation from the driving model over mountain areas, though it is considered a reasonable scheme for coarse resolution of NCEP RSM. And it is a reasonable approach to solve this problem with horizontal diffusion on pressure surface by computing the variables on sigma coordinates through the coordinate transform. Though isobaric surface is not a 'true horizontal' surface, it is quasi-horizontal and the results from this paper indicate that it is a sufficient method to have well-behaved forecasts and simulations. Though Zängl (2002) advocated horizontal diffusion on 'true horizontal' surfaces, i.e. height surfaces, by interpolation instead of coordinate transform, the errors, due to the interpolation from sigma surface to height surface and the needs of extrapolation for low atmosphere over slopes of mountains, may be as large as those caused by numerical truncation from coordinate transform. Nevertheless, the assumption free, simple coordinate transform might be better when the resolution is increasing.

The cases were selected here arbitrarily by collecting from different RSM user groups, but each one has its characteristic and provides its diversity to test with the same method. And the erroneous flow motion, due to the horizontal mixing on sigma surfaces, is enhanced by the large-scale atmospheric conditions. In unstable condition, the warming portion due to the erroneous horizontal diffusion on sigma surfaces will be enhanced to have rainfall over the mountains to induce a convergence along the hills, as Taiwan case and North American case over Mexico and AZNM. In the stable condition, the cooling portion will grow and result a divergence to spread cold air downhill, or even further to flat oceans as Oahu case and North American case over Baja of California. The scheme and results shown here provide an example for improving horizontal diffusion on pressure surfaces for mesoscale modeling.

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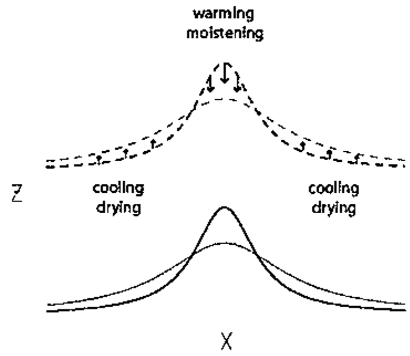

Fig. 1 A schematic plot to show the differences in height of the same sigma layers due to the model terrain differences between the outmost coarse resolution model (heavy solid and dashed curves) and inner fine resolution model (light solid and dashed curves). The solid curves indicate the model terrain heights and the dashed curves indicate the model layer heights. The arrows indicate the tendency of the sigma layer movement after applying horizontal diffusion on sigma surfaces.

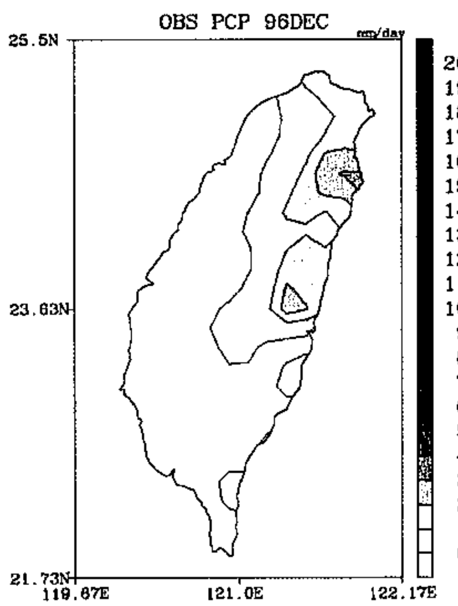

Fig. 2 Observed monthly averaged rainfall, in mm/day with contour interval of 1 mm/day, for

December 1996.

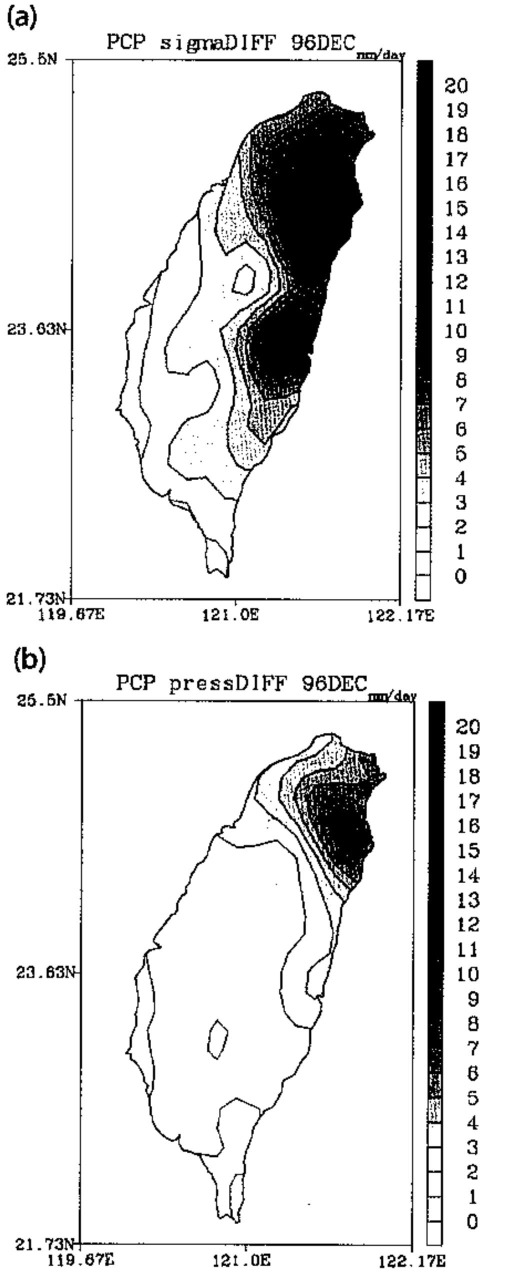

Fig. 3 Monthly averaged rainfall, in mm/day with contour interval of 1 mm/day, for December 1996 from NCEP RSM with (a) perturbation fourth-order horizontal diffusion on sigma surface and (b) full-field second-order horizontal diffusion on pressure surfaces.

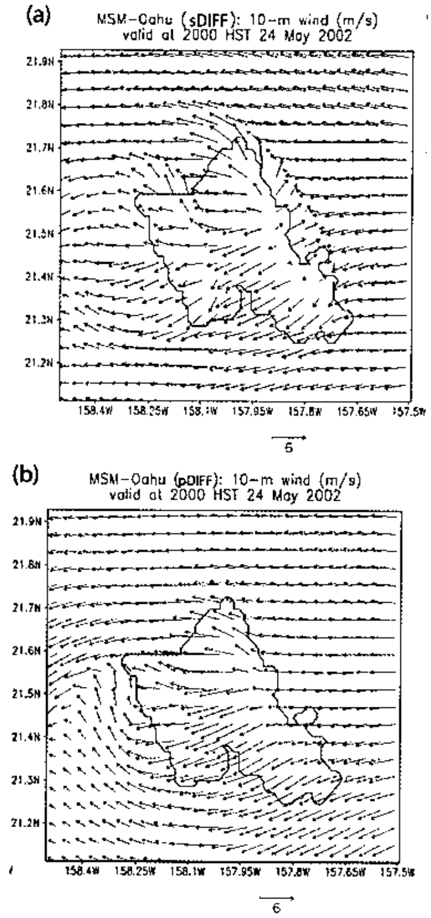

Fig. 4 6-hr forecast 10-m wind field in m/s valid at 2000 HST 24 May 2002 from (a) the experiment with perturbation fourth-order horizontal diffusion on sigma surfaces and with full field second-order horizontal diffusion on pressure surfaces, (b) and from the experiment with full field second-order horizontal diffusion on pressure surfaces.

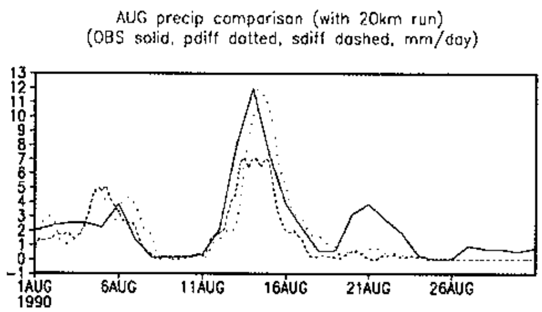

Fig. 5 The daily rainfall amounts during August 1990 over the states of Arizona and New Mexico from observation in solid curve, the experiment with full-field second-order horizontal diffusion on pressure surfaces in dotted curve, and the experiment with perturbation fourth-order horizontal diffusion on sigma surfaces in dashed curve.