Tilting and Rainwater Budget for Cumulus Updrafts

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1. INTRODUCTION

To parameterized the effects of convective downdrafts on large-scale fields, Cheng (1989a) presented a combined updraft-downdraft spectral cumulus ensemble model based on the rainwater budgets for both updrafts and the associated downdrafts. The model includes the budget of vertical momentum for each subensemble. The rainwater generated in the updraft is assumed to fall partly inside the updraft and partly outside the updraft. The mean tilting angle, which determines this partition, is estimated by considering stable statistically-steady states with random perturbations on the cloud-scale horizontal velocity. The vertical velocity and thermodynamic properties of the associated downdraft are then calculated considering the effects of rainwater loading and evaporation.

This combined updraft-downdraft model has been applied to diagnostic studies of the effects of convective downdrafts and mesoscale processes on the heat and moistur budgets of tropical cloud clusters (Cheng, 1989b; Cheng and Yanai, 1989). This model has also been incorpotated into the Arakawa-Schubert cumulus parameterization (Arakawa and Schubert, 1974). Preliminary results from semi-prognostic tests and GCM experiments using the modified Arakawa-Schubert parameterization which includes the convective downdrafts are very encouraging (Cheng and Arakawa, 1990a, b).

Cheng's model in its original form does not allow the detrainment of rainwater through cloud top due to the no-rainwater upper boundary condition for solving the rainwater budget equation. However, there is evidence that the rainwater detraining from cloud top can be an important source for anvil precipitation in a mesoscale convective system (e.g., Smull and Houze, 1987;

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Rutledge and Houze, 1989). The anvils and the associated stratiform precipitation are observationally significant features. They are potentially very important in terms of radiation calculations in numerical models.

This paper summarizes a revision of Cheng's tilting updraft model currently in progress. Major changes are a revised vertical momentum budget equation and a new upper boundary condition for the rainwater mixing ratio. In section 2 the rainwater budget equation and the vertical momentum budget equation used in Cheng (1989a) are reviewed. Alternative approach is discussed. In section 3 the stationary solutions of the rainwater budget equation for a range of the updraft tilting angle are shown and their stabilities with respect to small perturbation in the rainwater mixing ratio are analyzed. The stability analysis shows that there may be two convective regimes, one of which is associated with relatively small tilting angles and the other is associated with relatively large tilting angles. A simple time dependent version of the tilting updraft model is also carried out to illustrate the characteristics of the instability. In section 4 the existence of multiple convective regimes as suggested by the solutions of the rainwater budget equation and that revealed in an analysis of the cloud kinetic energy budget equation (Arakawa and Xu, 1990) are compared and their implications are discussed. Followed in section 5 is a summary and conclusions.

2. RAINWATER BUDGET EQUATION FOR THE UPDRAFT

Following Arakawa and Schubert (1974), we assume that an ensemble of cumulus clouds is statistically steady for a given large-scale environment although individual clouds are not. Updrafts are classified into subensembles according to their levels of detrainment. Using the vanishing-buoyancy condition at cloud top, the vertical profiles of the normalized mass flux and moist static energy are determined for each subensemble. Then, since the updraft is assumed to be saturated, the temperature and water vapor mixing ratio of the updraft can be obtained for each subensemble.

The rate of rainwater generation is parameterized using an autoconversion coefficient. Assuming that raindrops fall relative to the surrounding air with their terminal velocity, the rainwater budget equation for the updraft is derived from the balance between the generation of rainwater, the vertical convergence of in-cloud rain flux and the amount of outgoing rain flux.

Following Cheng (1989a), the rainwater budget equation for the updraft can be written as

$$\frac{1}{\cos\Theta}\frac{\partial F}{\partial t} = G - \frac{\partial H}{\partial z}, \qquad (1)$$

where

$$F \equiv mq_r\cos\Theta/w_c , \qquad (2)$$

$$G \equiv -\frac{2f_2}{\pi a} mq_r V_t \sin\Theta/w_c + mc_o\ell \qquad (3)$$

and

$$H \equiv mq_r (f_3 - V_t\cos^2\Theta/w_c). \qquad (4)$$

In the above expressions, m is the updraft mass flux, q_r is the rainwater mixing ratio, w_c is the updraft vertical velocity, V_t is the mean terminal fall velocity of raindrops, Θ is the updraft tilting angle, a is the radius of the updraft, C_o is the autoconversion coefficient for rainwater conversion from cloud water, ℓ is the cloud water mixing ratio, f_2 and f_3 are constants. The last term on the right-hand side of (3) is the generation rate of rainwater due to the autoconversion process.

The mean terminal fall velocity of raindrops is calculated using

$$V_t \equiv 36.34(\hat{\rho}q_r)^{0.1364}(\hat{\rho}/\rho_o)^{-0.5}, \quad ms^{-1} \qquad (5)$$

where $\hat{\rho}$ is the density of the updraft air, ρ_o is the air density of a reference state at the ground level (Soong and Ogura, 1973). To calculate the updraft velocity, we may use the vertical momentum budget equation which is written as

$$F_1(1+\gamma)[\frac{\partial}{\partial z}(mw_c) + m\lambda_d w_c] = \frac{m}{w_c}(B - gq_r), \qquad (6a)$$

where λd is the fractional rate of lateral detrainment, B is the thermal buoyancy including the loading of cloud water, g is the acceleration of gravity, γ is the virtual mass coefficient (Simpson and Wiggert, 1969), and f_1 is a constant. Eq.(6a) can be written in advection form as

$$\frac{\partial}{\partial z} w_c^2 + m(\lambda + \lambda_d)w_c^2 = \frac{B - gq_r}{f_1(1+\gamma)}, \qquad (6b)$$

where $\lambda \equiv (1/m)\partial m/\partial z$ is the fractional rate of entrainment. For a given vertical profile of the net buoyancy, $B-gq_r$, Eqs.(6a) or (6b) can be solved for the corresponding vertical velocity profile with a proper boundary condition of w_c at cloud base.

We are interested in the stationary solutions of the rainwater budget equation, i.e., the solutions of (1) with $\partial F/\partial t=0$. Because rainwater may fall through the cloud base and be carried away from the cloud top by the updraft air, we have to use "open" boundary conditions at both the cloud base and cloud top for numerical solutions of the rainwater budget equation. Because the different boundary conditions for the rainwater budget equation and for the vertical momentum budget equation, it requires some form of iterations for obtaining a solution.

To simplify the procedures for solving the rainwater budget equation, Cheng (1989a) utilizes the approximation

$$\frac{1}{mw_c}\frac{\partial mw_c}{\partial z} = 2\lambda - \frac{1}{\hat{\rho}}\frac{\partial \hat{\rho}}{\partial z} - \frac{1}{\sigma}\frac{\partial \sigma}{\partial z}$$

$$\approx 2\lambda - \frac{1}{\hat{\rho}}\frac{\partial \hat{\rho}}{\partial z}, \qquad (7)$$

where $m \equiv \sigma\hat{\rho}w_c$ has been used and σ is the fractional area convered by the updraft. Using (7), we can derive from (6a)

$$w_c^2 = \frac{B - gq_r}{f_1(1+\gamma)[2\lambda - \frac{1}{\hat{\rho}}\frac{\partial \hat{\rho}}{\partial q}]}, \qquad (8)$$

which is a local diagnostic relation between the vertical velocity, w_c, and the rainwater mixing ratio, q_r. Cheng (1989a) uses (8) for calculating the vertical velocity. When (8) is used, no iteration is required between solving for q_r from the rainwater budget equation and for w_c from the vertical momentum budget equation.

3. STATIONARY SOLUTIONS AND THEIR STABILITIES

Since the cloud-scale horizontal momentum budget equation are not used, the updraft tilting angle, Θ,Θis not explicitly calculated. Instead, we assume that each subensemble updraft has a constant tilting angle. If Θ is given, the rainwater budget equation can be solved in conjunction with the vertical momentum budget equation, i.e., one equation from (6a), (6b) or (8).

If (8) is used, the boundary condition q_r=0 at cloud top is required because B=0 at cloud top is an basic assumption for the one-dimensional cloud model. Clearly, no detrainment of rainwater from cloud top is allowed in this case. Examples of the stationary solutions for the rainwater budget equation with the vertical velocity determined by (8) are given by cheng (1989a,b).

In this section, we will examine the stability of the stationary solutions for the rainwater budget equation with open boundary conditions at cloud top and cloud base. The corresponding vertical velocity is calculated using (6b) with w_c = 0.1 ms^{-1} at cloud base in the following presentation.

Figure 1 shows the stationary solution of the rainwater budget equation for a range of the tilting angle, Θ, from 0° to 60°. This particular example is taken from cloud type 4, whose top reaches 160 mb, using GATE Phase III mean data [see Cheng (1989b)] as inputs. In this figure we see that the vertical profiles of rainwater mixing ratio, vertical velocity and in-cloud rain flux change smoothly as Θ increases from 0° to 28° and from 30° to 60°. There is a clear dis-continuity of the solution between 28° and 30°. There could be multiple stationary solutions for some values of Θ. However, the two groups of solutions separated by a dis-continuity between 28° and 30° have continuous characteristics through out a relatively large range of Θ. In this preliminary analysis of the properties of the rainwater budget equation, the problems of multiple solutions for some Θ are ignored.

For convenience, we call the two groups of solutions separated by a dis-continuity by solutions of small tilting angle (0° to 28°) and solutions of large tilting angle (30° to 60°), respectively. In general, the solutions for small tilting angle are characterized by a relatively small vertical velocity and a very small detrainment of rainwater from cloud top. On the other hand, the solutions for large tilting angle are characterized by a relatively large vertical velocity and a very significant detrainment of rainwater from cloud top. At 30° there detrainment of rainwater from cloud top account for about 40% of the total rainwater generated in the updraft. The detrainment ratio increases as the tilting angle increases. For comparison, the maximum ratio between the cloud-top detrainment and total generation of rainwater shown in the solutions for small tilting angle is less than 3%.

To examine the stability of the solutions shown in Fig.1, we linearized Eq.(1) with respect to small perturbation of q_r from the stationary solution. The linearized equation, which is consistent with the scheme used to solve for the stationary solution, has the form.

$$\frac{\partial}{\partial t} [q_r'] = [A] [q_r'] , \qquad (9)$$

where $[q_r']$ is a column vector and $[A]$ is a coefficient matrix.

Figure 2 shows the maximum real part of the eigenvalues of the coefficient matrix, [A], for various tilting angle ranging from 0° to 60°. There is a dis-continuity in the maximum values of the real part of the eigenvalues between 20° and 30°. Comparing Fig.2 to Fig.1, we see that the solutions for small tilting angle is unstable, while the solutions for large tilting angle is stable.

To visualize the characteristics of instability for the solutions of small tilting angle, we perform time integration of Eq.(1) at Θ=28° with initial conditions including i) q_r= 0.00001g/g, ii) q_r taken from the stationary solution at 20°, and iii) q_r taken from the stationary solution at 30°. The time integrations are terminated when the rainwater mixing ratio becomes too large that the updraft cannot maintain its upward vertical velocity. Fig.3 shows time-height sections of the rainwater mixing ratio from the time integrations of the rainwater budget equation with various initial conditions. Fig.3a shows that with initial condition i) the time integration is terminated after near 40 minutes. At near 30 minutes, we already see that q_r begins to increase abnormally. Fig.3b shows that with initial condition ii) perturbations begins to grow rapidly only slightly ahead of 50 minute mark. Fig.3c shows that from the first time step, q_r increases abnormally near cloud top and the time integration eventually terminated at near 15 minutes.

In addition to the time integrations for Θ=28°, we also perform time integration of the rainwater budget equation for Θ=30° with the same initial conditions as those for Θ=28°. The time integrations for Θ=30° (not shown) reaches a steady state, which is identical to that shown in Fig.1 at 30°, within one hour for all three initial conditions.

4. MULTIPLE CONVECTIVE REGIMES

Under very weak or no vertical shear environment, cumulus clouds may be unstable under the load of rainwater as characterized by the solutions for small tilting angle shown in Figs.1 and 2. In this convective regime, individual clouds will tend to be transient during their life cycles. The convection as a whole would tend to be dis-organized because of the random feature of the perturbation due to the instability of individual clouds.

Figure 3 indicates that instability is a tendency for the rainwater mixing ratio, q_r, to increase and terminate the updraft velocity. In reality, as q_r increases and the vertical velocity decreases, the tilting angle will increase and hence the rainwater may detrain relatively more efficient from the cloud and decrease the loading effect of rainwater. Therefore, there is a negative feedback from the change of the tilting angle to the growth of perturbation in the rainwater mixing ratio.

To see the effects of the feedback from the tilting angle on the time integration of the rainwater budget equation, we use the definition

$$\Theta \equiv \tan^{-1}[\frac{u_c}{w_c}] \ , \qquad (10)$$

where u_c is the mean cloud horizontal velocity relative to the motion of the cloud. We assume that the vertical profile of u_c would give the tilt of a cloud nearly constant in the vertical if w_c of the cloud is identical to the stationary solution shown in Fig.1b. Then, u_c can be estimated by inverting (10) and using a profile of w_c shown in Fig.2b at a given tilting angle. As a qualitative demonstration, we choose $\Theta = 20°$, which is not very small and not very close to the critical tilting angle between 28° and 30°.

With the given profile of u_c and Θ determined by (10), we integrate the rainwater budget equation in time starting with q_r=0.001g/Kg. The time-height sections of q_r and w_c from this time integration are shown in Figs.4a and 4b, respectively. We see that the solution tends to oscillate in time with a period of about 25 minutes. Excluding the initial 20 minutes or so, the magnitude of the updraft velocity is comparable to those of the solutions for small tilting angle and smaller than those of the solutions for large tilting angle. The detrainment of rainwater from cloud top (not shown) is small in this case.

If the environmental vertical shear is strong enough, cumulus clouds in such an environment may be characterized by the solutions for large tilting angle shown in Fig.1. In this convective regime, clouds of the similar cloud top height may tilt relatively uniformly, which could be a favorable condition for cloud to organized into mesoscale cloud clusters. Also, clouds in this regime will tend to have relatively large vertical velocity and, very likely, there will be a significant amount of rainwater detrains from cloud top which may contribute to the formation of anvils. Since there is a dis-continuity between the solutions for small tilting angel and the solutions for large tilting angle, the transition from one convective regime to another is expected to be abrupt and is not a linear function of vertical shear.

The existence of multiple convective regimes has an important implication on the cloud work function quasi-equivalence (Arakawa and Schubert, 1974). This can be illustrated by the simple mathematical model for the time-evolution of cumulus activity presented by Arakawa and Xu (1990). This model is based on the cloud kinetic energy equation which may be written as

$$\frac{dK}{dt} = AM - cK , \qquad (11)$$

where K is the cloud-scale kinetic energy, A is the cloud work function of the cloud kinetic energy generation per unit mass flux, M is the cumulus mass flux, and the last term on the right-hand side represents dissipation.

The time derivative of the cloud work function may be written as

$$\frac{dA}{dt} = -kM + F , \qquad (12)$$

where the right-hand side terms represent the adjustment due to cumulus process and the large-scale forcing, respectively. Since K is proportional to the square of the cloud velocity while M is proportional to the cloud velocity itself, we may assume

$$K = \alpha M . \qquad (13)$$

Using (12), (13) and treat α as a constant, the second time derivative of for M can be derived from (11), which gives

$$\frac{d^2M}{dt^2} + \frac{c}{2}\frac{dM}{dt} + \frac{k}{2\alpha}M = \frac{1}{2\alpha}F \qquad (14)$$

Eq.(14) is a damped oscillation equation for M. A solution of (14) for a given F is illustrated in Fig.5. Note that in the limit $\alpha \to 0$ we have $kM \to F$ and $dA/dt \to 0$, which is a perfect cloud work function equilibrium.

We expect from (13) that the larger the vertical velocity is for a fixed value of mass flux, the larger the α will be. The opposite is also true. Therefore, the convective regime characterized by the solution for small tilting angle would exhibit a better cloud work function quasi-equilibrium with the large-scale forcing than the other convective regime does.

Arakawa and Xu (1990) examine the relationship between the cumulus activity and the large-scale forcing under different vertical wind shear by comparing the precipitation rates with the large-scale forcing observed in numerical simulations of cumulus ensembles. Fig.6 shows the ensemble-average surface precipitation and the prescribed large-scale forcing (advections) from two experiments using a prognostic cumulus ensemble model [see Arakawa and Xu (1990) for the details]. Experiment Q03 does not have vertical shear imposed in the model, while experiment Q02 has a geostrophic wind with vertical shear typical of the environment for GATE convection. Otherwise, the two experiments are identical. We see in Fig.6 that the case with no vertical shear is close to the situation when $\alpha \to 0$, while the case with vertical shear is closely resemble to the illustration shown in Fig.5. The difference between the two experiments shown in Fig.6 is consistent with the expectation for the two convective regimes corresponding to the solutions for small tilting angle and those for large tilting angle, respectively.

5. SUMMARY AND CONCLUSIONS

The vertical momentum budget equation used in Cheng's (1989a) combined updraft-downdraft model has been revised and a new upper boundary for the rainwater mixing ratio, q_r, is imposed for numerical solutions of the rainwater budget equation. These changes allow the inclusion of the detrainment of rainwater from the updraft part to the anvil of a convective system. Both time-dependent and stationary solutions of the revised tilting upfraft model are studied for a range of the updraft tilting angle, .

We found that there is no stable stationary solutions of the rainwater budget equation if Θ is too small. The solutions in the unstable range, when the updraft tilting angle is small, are characterized by relatively small vertical velocity and no significant detrainment of rainwater from cloud top. On the other hand, the solutions in the stable range, when the updraft tilting angle is relatively large, are characterized by relatively large vertical velocity and a significant detrainment of rainwater from cloud top.

In a no shear or weak shear environment, cumulus convection may fall into a regime that can be characterized by the solutions for small tilting angle in the unstable range. Cumulus clouds in this convective regime will tend to be scattered and be transient through out their life cycles. In a sheared environment, on the other hand, cumulus convection may fall into a regime that can be characterized by the solutions for large tilting angle in the stable range. Cumulus cloud in this regime will tend to have a relatively uniform orientation and hence favorable for organized into mesoscale systems.

Observations shows that cumulus convections are often organized into mesoscale cloud clusters, suggesting that cumulus clouds may often be characterized by the solutions for large tilting angle. Furthermore, the vertical velocity, which has a negative feedback on the tilting angle, is large for large tilting angle in the stable range (Fig.1b), a large increase of u_c may increas the tilting angle only by a small fraction. Therefore, the optimal guess for the updraft tilting angle sould be somewhere in the stable range and near the critical angle separeting the stable range from the unstable range. For computational convenience, it is then reasonable to estimate the tilting angle for an updraft subensemble by choosing the minimum tilting angle in the stable range.

Cumulus clouds in the stable range will tend to have a relatively large vertical velocity. According to the analysis of a simple mathematical model for the time-evolution of cumulus activity, cumulus clouds with large vertical velocity would tend to enhance cloud work function nun-equilibrium. Some prognostic equation may eventually be introduced in a more advanced cumulus parameterization. In such a status, the present updraft model could provide a consistent description for the properties of cumulus subensembles under various equilibrium conditions.

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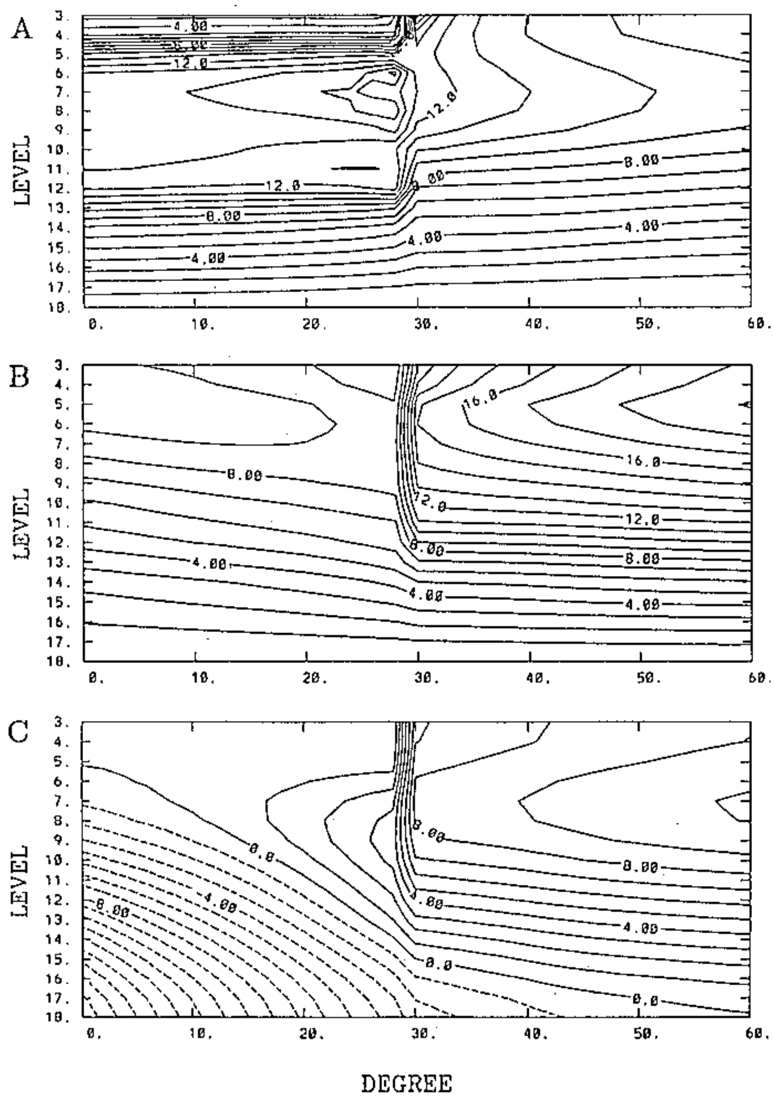

Fig. 1 Stationary solutions of the rainwater budget equation for θ from 0° to 60°. (A) q_r (g/kg), (B) w_c (ms^{-1}), and (C) in-cloud rain flux (arbitrary units).

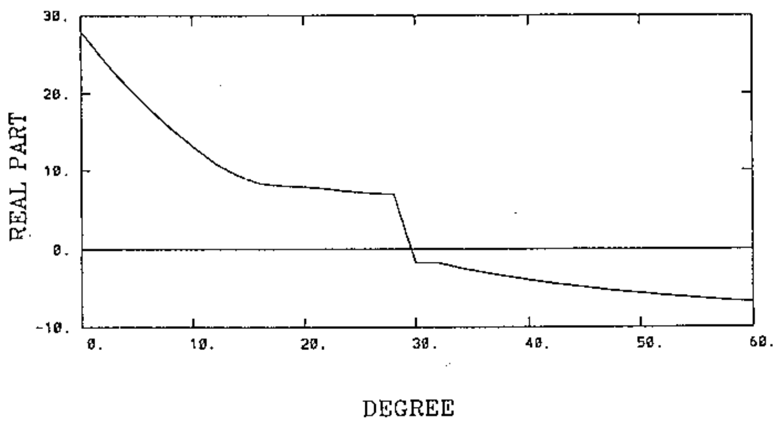

Fig. 2 Maximum real-part of the engenvalues of the linearized rainwater budget equation as a function of θ. Units: s^{-1}.

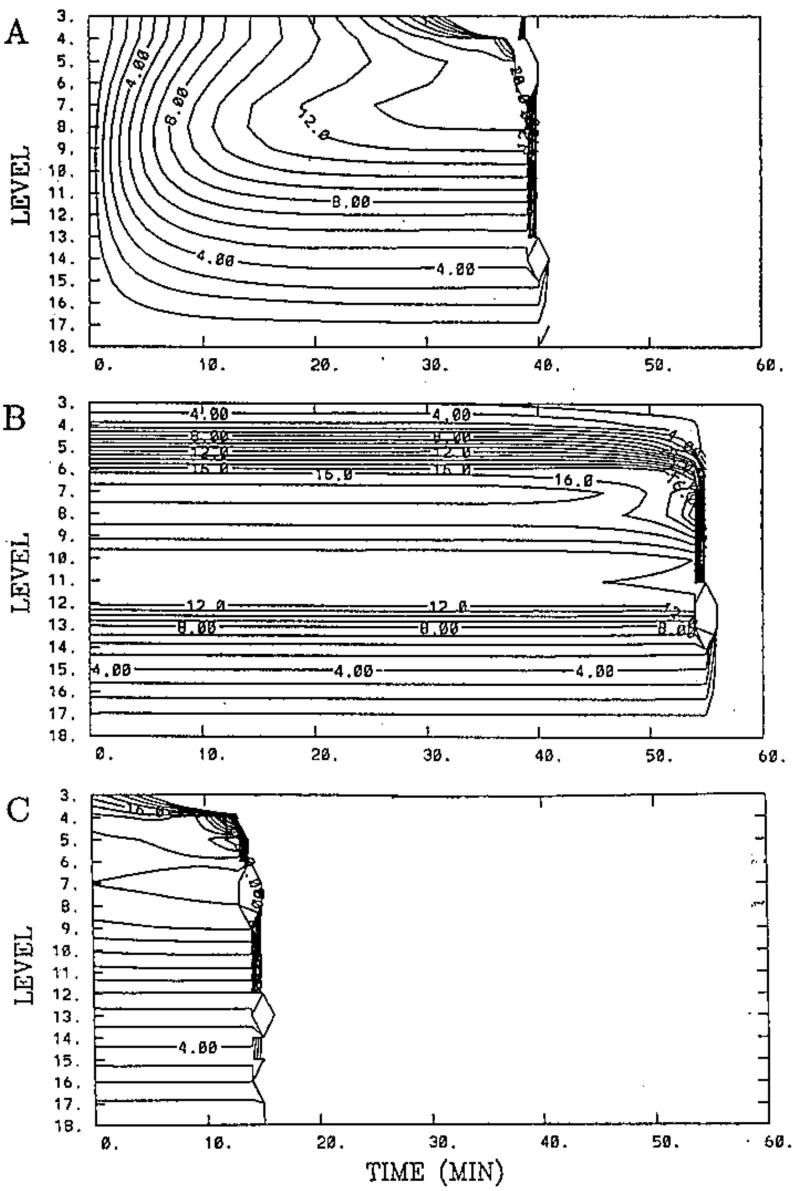

Fig. 3 Time-height sections of q_r from time integrations of the rainwater budget equation with initial conditions: (A) q_r=0.001g/kg, (B) q_r taken from Fig.1 at 28°, and (C) q_r taken from Fig.1 at 30°.

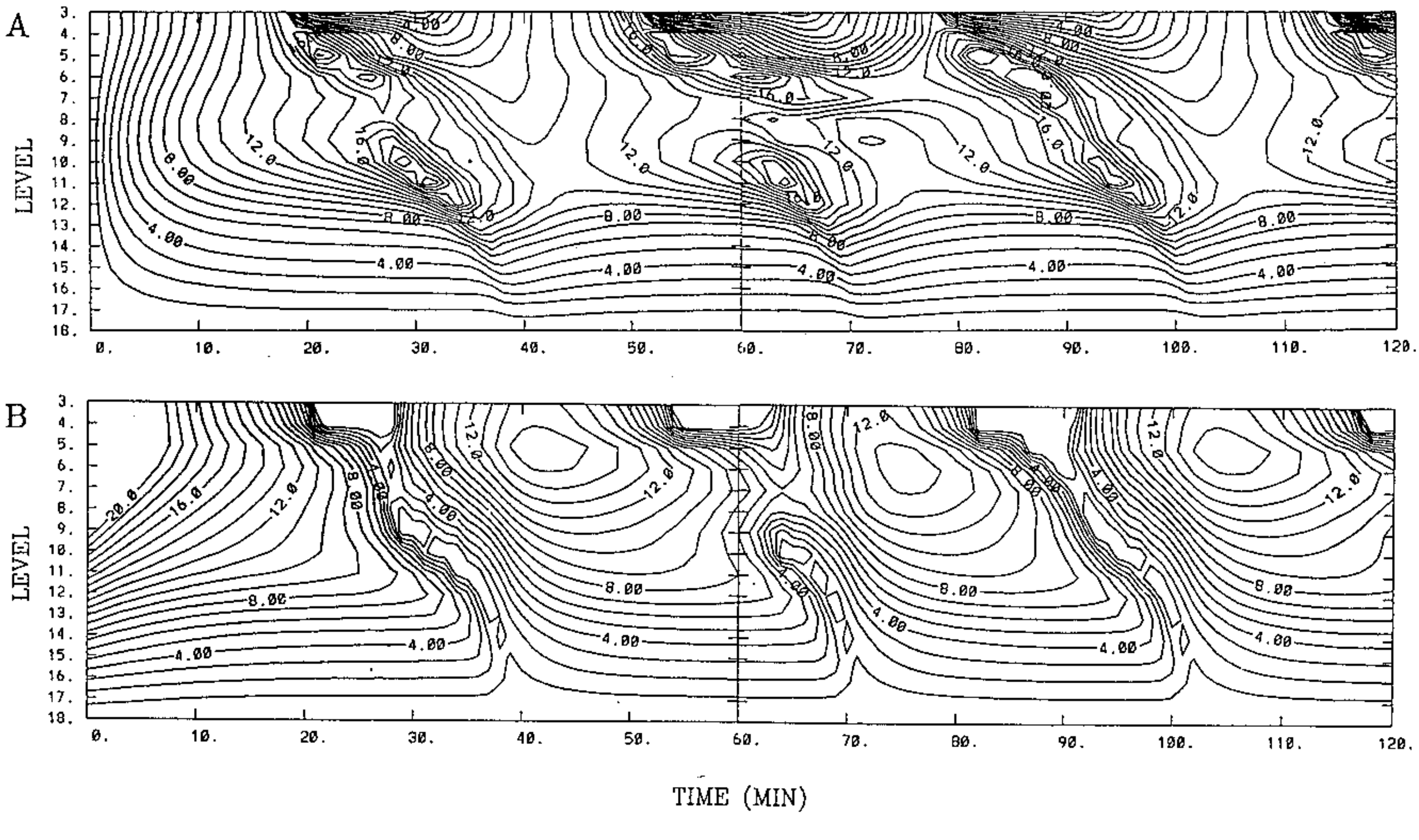

Fig. 4 Time-height sections of (A) q_r (g/kg) and (B) w_c (ms^{-1}) from time integration of the rainwater budget equation with a given u_c-profile.

Fig. 5 An illustration of the solution of Eq.(14) (thin line) for a given forcing (heavy line). [From Aramawa and Xu (1990)]

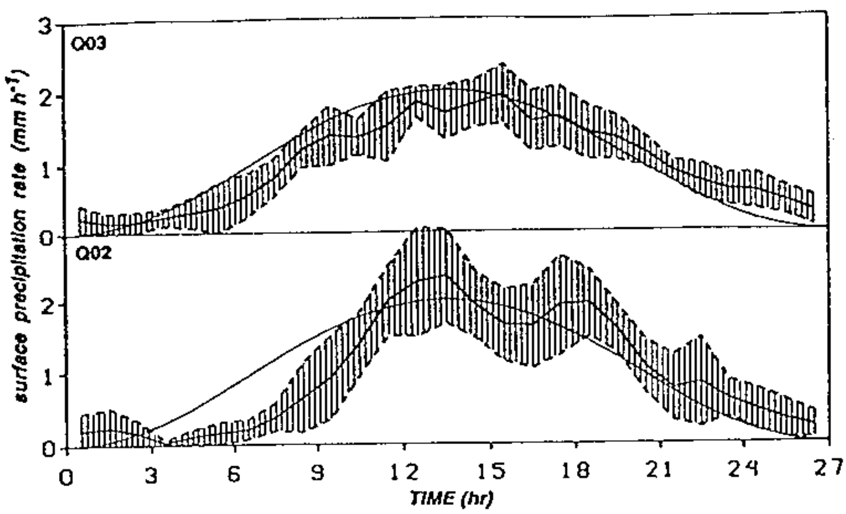

Fig. 6 The ensemble-average surface precipitation rate for the entire domain (thick solid lines) and associated standard deviation (error bars) from numerical simulations of cumulus ensembles. The abscissa is the phase of the imposed large-scale advective process. Thin solid lines show the imposed large-scale advective process with an arbitrarily chosen amplitude. [From Arakawa and Xu (1990)]

積雲的傾斜度及降水收支

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摘 要

積雲下冲流的產生和積雲降水密切相關，爲了參數化此下冲流的熱力學作用，我們首先須對積雲的降水收支有所認識。

定態的積雲降水收支乃是降水產生、雲內降水通量輻合、及降水捲出的平衡，而積雲的傾斜度則決定此平衡狀態下的降水分佈。我們發現不同的積雲有其特定的臨界傾斜角。當積雲的傾斜度大於其臨界角時，降水收支方程的定態解是穩定的。此穩定解通常有較小的水滴混合比及較大的垂直速度，在雲頂且有顯著的降水捲出，顯示砧狀雲發展的可能性。當積雲的傾斜度小於其臨界角時，降水收支方程的定態解是不穩定的。此不穩定解通常有較大的雨滴混合比及較小的垂直速度，在雲頂則無降水捲出。這兩種不同特性的解隱示積雲和大尺度環境間有多樣的平衡關係。