地球-電離層槽穴中極低頻波之非正交

Nonorthogonality between ELF Modes in the Earth-Ionosphere Cavity

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I. INTRODUCTION

For extremely low frequency (ELF) wave propagation in the Earth-Ionosphere cavity, the problem is usually treated by a residual mode theory, which has the advantage that only the lowest zero-order mode exists, whereas the higher non-zero-order modes can be neglected due to fast attenuation (Wait, 1970, p. 164; Galejs, 1972, p. 243). The existing mode theory is also based on the assumption that the surface impedances at the lower Ionosphere boundary were equal for all modes, or equivalently, all modes were considered to be orthogonal (Wait, 1970, p. 159; Galejs, 1972, p. 81).

Since nonorthogonality between ELF modes has never been studied, this report is to revise the ELF mode theory for the more realistic nonorthogonal case. Also derived is a rigorous mathematical criterion, which determines the significance of all ELF modes as compared with a designate mode.

As a first insight of the problem, only an ideal homogeneous and isotropic Ionosphere is considered; and the Earth is assumed to be perfectly conducting.

In Section 2 the present mode theory is reviewed. In Section 3 the theory is revised for nonorthogonal modes and the criterion to determine the significance of modes is established. Finally, numerical results are presented with discussions followed.

II. REVIEW OF MODE THEORY

To formulate the problem of ELF propagation, a spherical cavity is considered, which is bounded below by the Earth and above by a homogeneous isotropic Ionosphere. The cavity is assumed to be vaccume with permittivity ϵ and height h. The Earth with radius a is characterized by conductivity σ_e and permittivity ϵ_e while the Ionosphere has conductivity σ_i and it is impossible to consider the nonorthogonality properties. This is easily seen form (39) and (41). If more than one mode is significant, then it is not necessary to introduce such an extra insignificant mode.

From Fig. 9 it is also noted that the relative errors do not depend on θ. Since E_r of the $n=1$ mode is extremely small as compared with that of the $n=0$ mode, from (42) it is seen that the difference in $|E_r|_0$ and $|E_r|_n$ is due to different values of D_{ν_0}, which, as obtained from (41) is independent of θ. The independence of the relative errors in θ may not be true if more than one mode is significant.

The excitation factor for the nth mode is defined as (Galejs, 1972, P. 93):

$$\Lambda_n = \frac{kh}{2} \frac{[z_{\nu_n}(ka)]^2}{I_{nn}} \tag{49}$$

By combining (26) with (49), it yields

$$\Lambda_n = 2kha \sin(\nu_n \pi) z_{\nu_n}(ka) D_{\nu_n} \tag{50}$$

(50) shows that the excitation factor is directly proportional to D_{ν_n}. Since only the lowest mode is signigicant, the difference between $|E_r|_0$ and $|E_r|_n$ is due to the difference in D_{ν_0}. Thus nonorthogonality will reduce the value of the excitation factor. This reduction is the same as the relative error in $|E_r|$.

REFERENCES

GALEJS, J., 1972: *Terrestrial Propagation of Long Electromagnetic Waves*, Pergamon Press, New York.
SOMMERFELD, A., 1949: *Partial Differential Equations in Physics*, Academic Press, New York.
WAIT, J. R., 1970: *Electromagnetic Waves in Stratified Media*, 2nd ed., Pergamon Press, New York.
WATSON, G. N., 1919: The transmission of electric waves round the earth, *Proc. Roy. Soc. London*, Ser. A, 95, 546-563.

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摘　　要

本報告導出一數學公式作爲比較因數，以決定在地球-電離層槽穴中，極低頻波各模型的重要性。在演算極低頻波的電場時，考慮模型的非正交性；並且由數值結果比較此電場與正交模型的電場的差別。雖然在此不同情形下都只有最低次的模型重要，但在計算非正交模型的電場時，必需引入衰減最小的高次模型。若一均勻及等向性的電離層導電係數爲 3.8×10^{-6} mhos/m 及 2.0×10^{-6} mhos/m，則在 3 KHz 正交模型會產生分別爲 3.4%及 4.4%的相對誤差。此誤差不隨波源與接收者之間的距離改變，但隨頻率之減低遞減。比較非正交和正交模型的電場也顯示出前者的激發因數較小，其差別和電場的相對誤差一樣。